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Impact and Applications of Fuzzy Cognitive Map Methodologies

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Abstract Since their introduction in 1986, Fuzzy Cognitive Maps (FCMs) have been comprehensively studied, applied, and extended with growing interest and are still expanding in use. This chapter discusses the impact of Fuzzy Cognitive Maps as a knowledge acquisition, knowledge reasoning and modeling methodology, on its own, and in synergy with other soft computing, computational intelligence and knowledge-based methodologies. It discusses the general structure and development of FCMs and their topologies as well as extensions to fill specific problem needs. The extensive application areas are also presented along with future research directions.

Keywords Fuzzy · Fuzzy cognitive maps · Soft computing

1 Introduction

In the real world, despite people's preference for precision and accuracy, information, variables and values are frequently estimated and they are characterized either as fuzzy or belonging to an interval [22]. Much attention is put on handling the characterization of a variable and not its precise value, in order to reach a conclusion, which has led to approaches such as Internal Analysis, Fuzzy Cognitive Maps (FCM) and others. Here, we focus on Fuzzy Cognitive Maps methodologies and their contribution in facing real world problems and cases [7, 8].

Fuzzy cognitive maps use fuzzy logic, a form of multi-valued logic in which the truth values of variables may be any real number between a range of numbers. This "logic" is closer to human representation since linguistic variables are often

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used when describing quantities of variables [47]. Thus, the values of such variables incorporate all the parameters that a characterization of a situation can have. It is employed to handle the concept of partial truth, where the truth value may range between completely true and completely false [23].

FCMs have gained considerable research interest due to their ability in representing structured knowledge and in modeling complex systems. Many researchers have carried out extensive studies on different aspects of FCMs. Generally speaking, Fuzzy Cognitive Maps (FCMs) is a soft computing technique used for causal knowledge acquisition and causal knowledge reasoning. FCMs' modeling approach resembles human reasoning; it relies on human expert knowledge for a domain, making associations in terms of generalized relationships between domain descriptors, concepts and conclusions [25]. FCMs model any real world system as a collection of concepts and causal relation among concepts.

We have gathered and illustrated the main keyword appearance in article titles, abstracts and keywords based on frequency of their reference. Based on that, Fig. 1 was inferred. It illustrates the main labeling and characteristics found in published Fuzzy Cognitive Maps related papers. The size of words shows the frequency of the corresponding keywords.

Figure 2 illustrates a more detailed representation of the main attributes referred to FCMs, which is the result of excluding the words of fuzzy cognitive maps and cognitive maps, as their presence frequency is omnipresent in existing papers, being so great, that do not let the remaining keywords be visible enough.

If someone examines Fig. 2, they will infer interesting conclusions about Fuzzy Cognitive Maps: (a) what are the most essential characteristics of this theory, (b) what kind of approach it is, (c) what is the research field that it belongs to, (d) with which other approaches it is complementary, (e) which is its main contribution, and (f) where it has been applied. Figure 2 could be considered as having high entropy because the names and the strength of the presented keywords have a great amount of information for the reader.

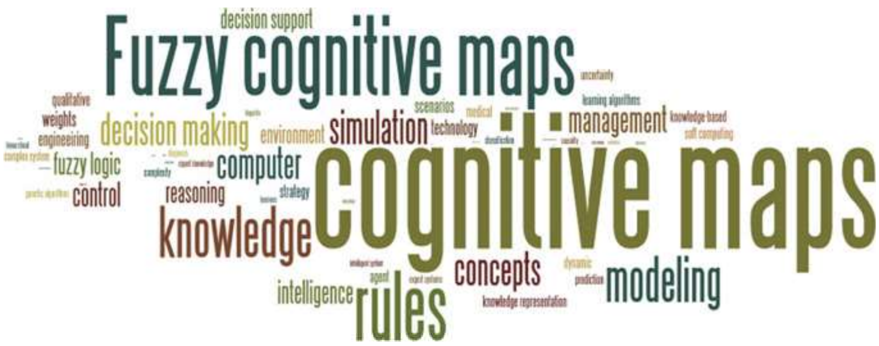


Fig. 1 Graphic presentation of keywords that characterize fuzzy cognitive maps according to their frequency presence

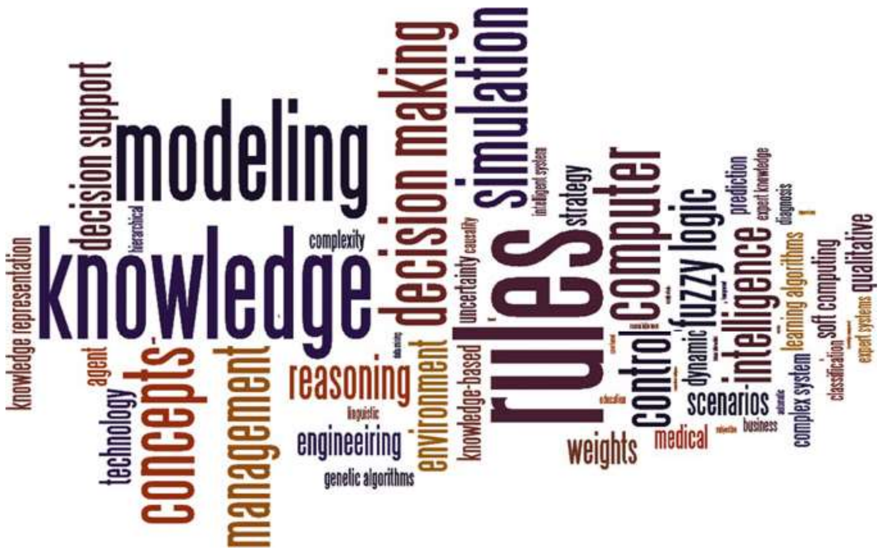


Fig. 2 A detailed Fig. 1, by excluding the words "fuzzy cognitive maps" and "cognitive maps"

In this chapter, we present information for the evolution of FCM methodology in order to review and discuss it critically. Actually, it describes the FCMs from its roots till today, it presents all the different FCMs methodologies that have been proposed and a comparison and evaluation of them. The ambition of this study is to inaugurate a further adoption and usage of Fuzzy Cognitive Maps and their extensions. We firstly refer to the increasing need for adaptable and efficient FCM approaches. Section 2 describes the generic structure of FCMs and then Sect. 3 presents designing of FCMs based on experts and improving FCM by learning. The main direction on generalizing FCMs is presented regarding topology/structure in Sect. 4. Section 5 presents synergies with other technologies while Sect. 6 discusses the extended applicability and usefulness of FCMs since their inception in various areas. Finally Sect. 7 concludes this chapter and proposes future research directions.

2 Generic Structure of FCM and Its Development

Fuzzy Cognitive Map (FCM) is a soft computing modeling technique, which originated from the combination of Fuzzy Logic and Neural Networks. At first, Axelrod [2] introduced Cognitive maps as a formal way of representing social scientific knowledge and modeling decision making in social and political systems. Later on Kosko [19] enhanced cognitive maps considering fuzzy values for them, introducing partial causality among concepts that allows degrees of causality and not the usual binary logic. A Fuzzy Cognitive Map describes a system in a one-layer network whose interconnected nodes are assigned concept meanings and the interconnection weights represent cause and effect relationship among concepts. The FCM approach is used for causal knowledge acquisition and representation; it supports the causal knowledge reasoning process and belongs to neuro-fuzzy systems that aim at solving decision making problems, modeling and control problems.

FCM is an illustrative causative representation for the description and modeling of any system. FCMs are dynamical, fuzzy signed directed graphs, permitting feedback, where the weighted edge w_{ij} from causal concept C_i to affected concept C_j describes the kind and amount by which the first concept influences the latter, as is illustrated in Fig. 3. Experts design and develop the structure of the system, including the "nodes" (i.e., concepts) that correspond to variables, states, factors and other characteristics that are used to model and describe the behavior of the system. They determine the network's interconnections, using linguistic variables to describe the relationships among concepts. Then all the proposed influences from experts are combined and aggregated and thus, the initial weights are determined. Next learning methods are introduced so that to ensure that the FCM will converge to an equilibrium point.

The weight of the arc between one concept and another could be positive ($W_{ij} > 0$), which means that an increase in the value of first concept leads to the increase of the value of the interconnected concept; and a decrease in the value of first concept leads to the decrease of the value of latter concept. When there is negative causality ($W_{ij} < 0$) an increase in the value of the first concept leads to the decrease of the value of the latter concept and vice versa. Finally, there may be no causality ($W_{ij} = 0$).

The value A_i of concept C_i expresses the degree of its corresponding physical value. FCMs are used to model the behavior of systems; during the simulation step, the value A_i of a concept C_i is calculated by computing the influence of the interconnected concepts C_j's on the specific concept C_i following the calculation rule:

$$A_i^{(k+1)} = f\left(\sum_{\substack{j \neq i \\ j=1}}^{N} A_j^{(k)} \cdot w_{ji}\right) \tag{1}$$

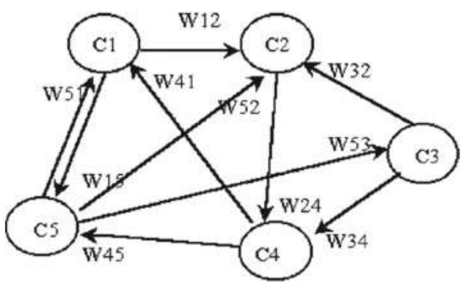


Fig. 3 The general FCM model

where $A_i^{(k+1)}$ is the value of concept C_i at simulation step $k+1$, $A_j^{(k)}$ is the value of concept C_j at simulation step k, w_{ji} is the weight of the interconnection from concept C_j to concept C_i and f is the sigmoid threshold function:

$$f = \frac{1}{1 + e^{-\lambda x}} \tag{2}$$

where $\lambda > 0$ is a parameter that determines its steepness. The sigmoid function is selected since the values A_i of the concepts have to in the interval [0, 1], where concepts take values.

Equation (1) does not take into consideration the possible memory of each concept, so the value A_i for each concept C_i is finally calculated by the following rule:

$$A_i^{(k+1)} = f\left(A_i^k w_{ii} + \sum_{\substack{j \neq i \\ j=1}}^{N} A_j^{(k)} \cdot w_{ji}\right) \tag{3}$$

It is mentioned that the model presented in Eq. 3 and illustrated in Fig. 4, is characterized by high memory abilities, especially in the case that $w_{ii} = 1$, because at every running step $k+1$ it is considered the total value of concept $A_i^{(k)}$ at step k. Especially, in the case that all the concepts have self-weights then the value of concepts is just slightly updated by the interconnected concepts.

One of the main strengths of Fuzzy Cognitive Maps was the introduction of linguistic variables. That means, the influence between concepts is described with linguistic weights, which according to the construction methodology are aggregated so that at the end a linguistic weight is inferred to describe the influence of one concept to the other. The overall linguistic weight using a defuzzification method is transformed into a numerical weight in the interval [−1, 1] [32].

Learning algorithms have been proposed for training and updating FCMs weights. Adaptation and unsupervised learning methodologies are used to adapt the FCM

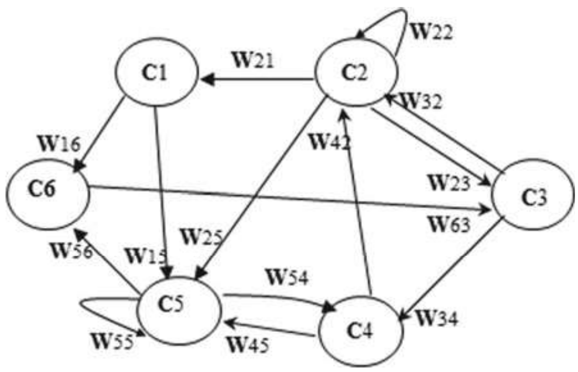


Fig. 4 The FCM including concept memory characteristics

model and adjust its weights. Kosko and Dickerson suggested the Differential Hebbian Learning (DHL) to train FCM, but without a detailed mathematical formulation or implementation at a specific problem [10].

The Differential Hebbian Learning (DHL) proposed unsupervised learning for the case of bivalent FCMs. The DHL law correlates the changes of two concepts, if value of concept C_i changes at the same direction with value of concept C_j (e.g. C_i increases when C_j increases), the edge strength w_{ij} between the two concepts is increased; otherwise the edge strength is decreased. At each time step *t*, the value for weight w_{ij}, the linkage between concept C_i and concept C_j, is given by the discrete version of the DHL law:

$$w_{ij}^{(k+1)} = w_{ij}^{(k)} + \mu_t(\Delta C_i^{(k)} \cdot \Delta C_j^{(k)} - w_{ij}^{(k)}) \tag{4}$$

where ΔC_i is the change in the value of *i*-th concept, in other words $\Delta C_i^{(k)} = C_i^{(k)} - C_i^{(k-1)}$. The learning coefficient μ_k decreases slowly over time, with the following equation [20]:

$$\mu_k = 0.1\left[1 - \frac{k}{1.1N}\right] \tag{5}$$

where the positive constant N ensures that the learning coefficient μ_k never becomes negative.

Then many researchers followed the same path and adapted Hebbian Learning algorithms for FCMs. A first attempt was the introduction of DHL approach [17], called Balanced Differential Algorithm (BDA). BDA seemed to work better in learning patterns and modeling a given domain than the classical DHL approach, but it worked only to binary FCMs.

Activation Hebbian Learning (AHL) and Nonlinear Hebbian Learning (NHL) are two unsupervised weight adaptation techniques that were applied on the basic FCM model [30, 31]. Both AHL and NHL have been introduced to fine-tune FCM causal linkages among concepts. Both algorithms successfully updated FCMs and led to establish FCMs as a robust technique that could further improve the good knowledge of a given system or process. They updated the initial information and experts' knowledge achieving to keep the values of output concepts within the desired bounds for the examining problem. These learning techniques contributed to the establishment of FCM as a robust technique, that can efficiently update the cause–effect relationships among FCM concepts and their effectiveness in real modeling problems [32]. New advanced algorithms have been proposed for FCM training with successful results as proved for the applications in different areas. Learning algorithms have been based on the basic Hebbian algorithm or come from other fields, such as genetic algorithms, swarm intelligence and evolutionary computation [27, 34].

3 Design FCMs Based on Experts

Initially, when Axelrod introduced cognitive maps in the 1970s for representing social scientific knowledge, he presented the adjacency matrix representation of cognitive maps. As cognitive maps were too binding for knowledge-based building, Kosko proposed Fuzzy Cognitive Maps introducing fuzziness for the general causality [19]. The knowledge acquisition is inherent in the approach of building cognitive maps but the fuzziness of the combined knowledge rises to the level of the fuzziest knowledge source. It presented the difference between the expert systems and the non-linear dynamical nature of FCMs. Kosko introduced the combining fuzzy knowledge networks and proposed the augmented FCMs, which comes from the combination of particular FCMs from different experts [20], while at the same time he proposed the unsupervised Hebbian learning for training FCMs. Knowledge base quality is hard to quantify and guidelines are elusive. In 1991 a new approach was proposed to take under consideration the credibility weight [47], where every expert has his own knowledge, experience and way of solve different problems.

A significant contribution on structuring Fuzzy Cognitive Maps, investigating and proposing a set of developing methodologies for FCMs based on human experts who use fuzzy rules to explain the cause and effect among concepts were introduced in the late 90's [42, 43, 45]. New mathematical descriptions of FCMs were also investigated along with their implementation for modeling and control complex systems [44, 46]. This put the Soft Computing technique of Fuzzy Cognitive Maps in the center of interest of a wide audience and thrust FCMs' investigation and application in a wide range [26].

A Fuzzy Cognitive Map (FCM) could be built by a group of experts, using an interactive procedure of knowledge acquisition. Every expert is asked to define the main concepts that should be present at the FCM based on his knowledge and experience on the operation of the system. A concept can be a characteristic, a state or a variable or input or an output of the system. An expert has in his mind a conceptual model of the system, which consisted of the main factors that are crucial for the modeling of the system and he represents each one by a concept. In addition to this, he has a subjective understanding on which elements of the system influence other elements; by which kind and to what degree. So he is able to infer regarding the negative, positive or zero effect of one concept on the others. Moreover, he is able to assign a linguistic value for each interconnection, since it is assumed that there is a fuzzy degree of causality between concepts.

To acquire the knowledge and experience of a group of experts the following methodology is applied. All experts are polled together and they determine the relevant factors, the main characteristics of the system and thus the concepts consisting the Fuzzy Cognitive Map. Then, each expert individually determines the structure of the FCM using fuzzy conditional statements to describe the relationship of one concept to the other. Every expert uses an IF-THEN rule to justify the cause and effect relationship among two concepts and infer a linguistic weight for each interconnection.

A fuzzy rule of the following form is assumed, where X, Y, Z are linguistic variables:

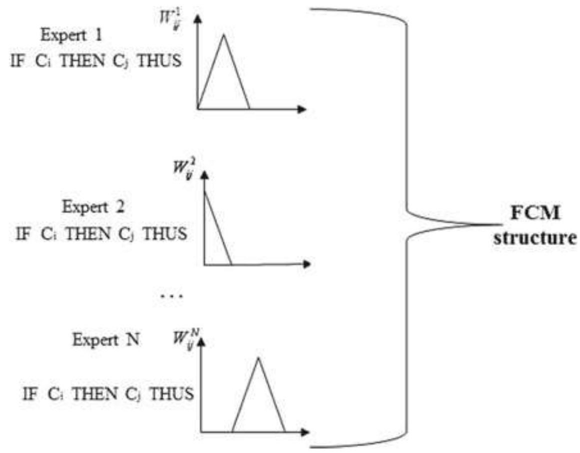
IF an X change occurs in the value of concept C_i THEN a Y change is caused in the value of concept C_j. Thus, influence of *concept C_i to concept C_j is Z.*

Thus, every expert is forced to infer a rule and to assign a linguistic value (weight) for the relationship between the two concepts. So the causal relationship is described by a fuzzy rule, which gives the grade of causality between concepts and so the corresponding weight is inferred. Then, the set of weights of each interconnection are integrated and a defuzzification method is used to produce a numerical weight for the interconnection. In fuzzy logic literature many methods for defuzzification have been proposed, such as the popular method of Center of Area, which is used here and the produced numerical weight will belong to the interval $[-1, 1]$.

As an example, the case where experts describe the relationship among two concepts is depicted at Fig. 5. Every expert describes the relationship among two concepts using a fuzzy rule and he infers a linguistic variable for the corresponding weight that then are all aggregated to describe the specific relationship resulting in the whole structure.

Novel integrated approaches on developing Fuzzy Cognitive Maps by combining human expert knowledge with existing recorded information and historical data have been proposed [41]. They combine the extraction of information from unstructured data, which is transformed into knowledge as a FCM along with exploiting the knowledge and expertise of experts by providing them with more information and supportive data, in the form of particular evidence-based information available in the literature in order to better justify their selections [26].

Fig. 5 The procedure to develop FCM structure



4 Generalization of FCM Topology and Design

The primary FCM model introduced by Kosko has been used as a basis, but new FCM generalized structures aiming to more computational effectiveness and objectiveness have been developed. It has been combined with other approaches to produce effective models that achieve better results for various applications in different areas. Additionally, computational methods and algorithms have been introduced that take advantage of historical data to create more dynamic FCMs models. Semi-automated methods require a relatively limited human intervention, whereas fully automated approaches are able to compute the FCM solely based on the historical data, that is, without human input [38, 39].

There are two main extensions, the first one includes the models that have been developed with interference to the basic FCM structure and are oriented to enhance the characteristics that affect the final result. They also use fuzzy sets and/or similar approaches and they can calculate new weights and elements and/or they try to measure the uncertainty and hesitancy. In addition to this, hybrid models have been developed that combine different technologies from different areas and they manage to make a more effective and realistic model, improving the characteristics of the basic FCM model [24]. A second generalization approach includes the basic FCM structure and the use of training algorithms to change and update their weights, leading to better and/or faster results.

4.1 Enhancement, Generalization of Individual Units and New Topologies (Architectures)

There is a series of enhancements and extensions to FCMs mainly based on various artificial intelligence techniques:

Rule Based Fuzzy Cognitive Map (RBFCM) is a standard rule based fuzzy system, where someone can add feedback and mechanisms to deal with causal relations [5, 6]. It consists of fuzzy nodes and fuzzy rules, which relate and link concepts. Each concept is permitted to have many membership functions that they represent either the concept's possible values or possible values of its change. The evolution of the system is iterative. The RB-FCM is an approach for modeling the evolution and stability of the entities that compound a domain of study. The RB-FCM simulates the system's dynamics from a qualitative and causal perspective.

The main characteristic of the RBFCM methodology is that it introduces a fuzzy operation, the Fuzzy Carry Accumulation (FCA), which accumulates the inferences to each concept from the other concepts and then based on the calculated result of effects allows the introduction or removal of rules among concepts in the existing model, making the system dynamic [5, 6, 9].

Dynamical Cognitive Network (DCN) is another extension to FCMs. Miao et al. introduced a mechanism that can quantify the description of concepts with the required precision and the strength of the causality among concepts [28]. This is the first model that separates the three fundamental elements of a causal system: the cause, the causal relationship and the effect. DCN was designed to enhance the dynamic aspect of the system as causal inference systems are dynamic in nature.

DCNs take into account the direction of the causal relationship, the strength of the cause, and the degrees of the effect. A general DCN describes not only the strength of causes, impacts, and effects, but also the dynamics of how the impacts are built up. DCNs tried to overcome the lack of time for FCMs by introducing the temporal concept. Miao et al. introduced the dynamic functions for the arcs to represent the dynamic and temporal effects of causal relationships. Later on, they referred to the transformation and succeeded equivalence between DCNs and FCMs, which makes easier the way that a designer familiar with FCM can use the simplified DCN [29].

Competitive Fuzzy Cognitive Maps (CFCMs) were introduced [11] for the use of FCMs in decision support. In CFCMs there are factor nodes (those that contribute to the decisions and interact with other factor nodes) and the decision nodes that accept inputs from factor nodes and "compete" with the other decision nodes using inhibitory (negative-valued feedback) in order to reach a single decision. These networks have been extensively used in medical decision support [40] when decisions are mutually exclusive, either in diagnosis or intervention planning.

Fuzzy Cognitive Networks (FCNs) introduced as an extension to the traditional FCMs [21]. The framework for this model consists of the representation level (the cognitive graph), the updating mechanism, which receives feedback from the real system and the storage of the acquired knowledge throughout the operation. Every node has its one label and they are characterized as control, reference, output, simple and operation nodes. But, it is possible a node to have more than one label. FCN reaches always an equilibrium point because it uses direct feedback from the node values and the limitations imposed by the reference nodes. The nodes of FCN take as input the desired values, which represent the goals that set for the system. Experts convey information related to the structure and the corresponding initial weights, thus the FCN system reaches an equilibrium point. The extracted decisions are applied to the real system and the feedback of the real system is transferred to the FCN model.

Intuitionistic FCMs (IFCMs) are based on Intuitionistic Fuzzy Sets (IFS), which enhance the FCM methodology by the introduction of hesitancy factors into the edge weights. IFS can be viewed as a generalization of fuzzy sets that may better model imperfect information as Intuitionistic Fuzzy Set (IFS) provides a mathematical model suitable for modeling the imprecision which is inherent to the real world problems. IFS is an extension of fuzzy sets introducing an additional degree, which is the degree of hesitancy (uncertainty). IFSs are comprised of elements characterized by memberships and non-memberships values [1]. IFCMs utilize intuitionistic fuzzy sets and reasoning for handling experts' hesitancy for decision making [18]. IFCMs use the intuitionistic reasoning, which adds the degree of hesitancy in the relationships defined by experts. In this way, the experts not only express the influence between two concepts, but also their hesitancy to express that influence.

Granular Fuzzy Cognitive Maps have been introduced by Pedrycz and Homenda [35] aiming to better capture the experimental data and facilitate the procedure of developing a Fuzzy Cognitive Maps utilizing several sources of knowledge. In these, the idea of granular connections that are updated following a supervised gradient based approach was introduced. They are characterized by the dynamic pattern of states and their propagation of information granularities. Also, a methodology to develop an overall aggregated Granular Fuzzy Cognitive Map was introduced.

A new extension was the introduction of time series in order to make the FCM a fully automated and autonomous system [36, 37]. This approach refers to the use of techniques of Granular Computing, such as fuzzy clustering to form concepts of well-articulated semantics. Pedrycz et al. introduced a mechanism which uses granules to represent numeric time series which in turn will give rise to the correspondent nodes of the FCM.

4.2 Timed Fuzzy Cognitive Maps

Often in modeling systems, phenomena, problems etc., time is a parameter that may play a vital role in the evolution of the outcome [49]. Timed Fuzzy Cognitive Map (T-FCM) includes the idea of time and bases the evolution of the cognitive map outcome on previous time units. This is different from the step by step convergence of a FCM; it actually inserts the concept of time parameter within the FCM itself taking into consideration how each parameter may change over time, both in value, as well as in importance to the outcome. The T-FCM also permits the user intervene on the overall procedure, by changing values during the time units, while the intermediate results illustrate the evolution of a case during the time. In order to define the weights between each interconnection and each time unit, the experts who design the T-FCM need to recall the progression of the phenomenon being simulated during the time. They should define the initial weights, denoting which concepts' dependencies (weights) have lower or higher influence during the progression of the case. The direction of this change depends on the contribution of the additional parameters that may be active or inactive during the progression of a case. That is, during the time the interdependencies among some factors have different degrees of influence compared to others and this change depends on both time and the additional parameters; interconnections can become weaker or stronger, while some of concepts may be deactivated and others activated. Therefore, a set of discriminator factors m_k are defined based on the activation or not of parameters. The learning method of the model is based on the basic FCM training with enhancements, in order to take into account the time unit and the individual characteristics of the under investigation model. Thus, in T-FCM the concept of time in the calculation of the next concept value was inserted and this time unit plays a significant role during the training. For the T-FCM the interconnections between concepts $d^{t}_{m,t,w_{ij}}$ are dependent on the weight w_{ij}, the case m_k and the corresponding time unit t [3].

Specifically, the value A_i of the concept C_i expresses the degree of its corresponding physical value. At each simulation step, the value A_i of a concept C_i is calculated by computing the influence of other concepts C_j's on the specific concept C_i on a specific time unit for a specific discrimination factor following the calculation rule (6). Thus,

$$A_i^{k+1}(t) = f\left(A_i^{k+1}(t-1) + \sum_{t=1}^{t-1}\sum_{\substack{j\neq i \\ j=1}}^{n} A_j^k(t-1)(d^t_{m,t,w_{ij}}), m\right) \tag{6}$$

where A_i^{k+1} (t) is the value of concept C_i at simulation step $k + 1$ for a time unit, A_j^k $(t - 1)$ the value of the interconnected concept C_j at simulation step k, $d^t_{m,t,w_{ij}}$ is the weight of the interconnection between concept C_j and C_i, and f is a sigmoid threshold function.

5 Synergies of FCMs with Other Methods for Improved Efficiency

Even though there have been a wide variety of statistical, soft computing and knowledge based methods used in synergy with FCMs for learning and convergence, there are situations where FCMs do not reach a distinct outcome, in models where a single outcome should result. In this section, two examples of extensions of FCM models presented in Sect. 4 using synergies with Case Based Reasoning and Hidden Markov Models are presented.

5.1 Competitive Fuzzy Cognitive Maps with Case Based Reasoning

As mentioned in Sect. 4 Competitive Fuzzy Cognitive Maps (CFCMs) have been used for Decision Support when decisions are mutually exclusive, which is often the case in medical diagnosis support [14]. However, there are situations where the CFCM does not converge to clearly distinct outcomes. This is the case when two or more outcomes in the CFCM converge to final values that do not differ by at least 10%. Since it is of critical importance to have a high degree of confidence in the decision reached, Case Based Reasoning (CBR) can be used to find cases that are similar to the particular case [12, 13].

Figure 6 diagrammatically shows the CBR enhanced CFCM Decision Support Model for medical applications. Here the relevant patient data is input to the CFCM and the factor concepts take their initial values from this input data. Patient information are experimental results, test results, physical examinations and other descriptions of symptoms and measurements of physical qualities. This information can be described either in numerical values or in fuzzy linguistic weights which are then

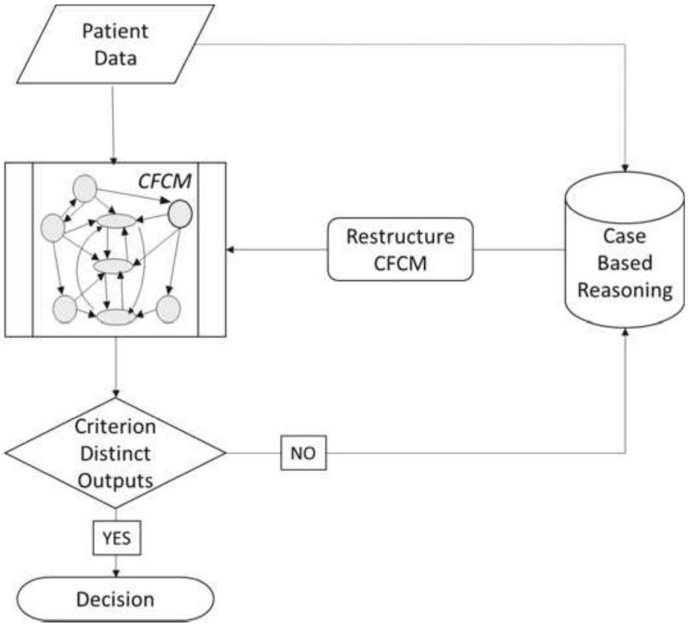


Fig. 6 CBR CFCM algorithm for improved convergence of MDSS

transformed into a numerical weight in the range [0, 1], i.e., the allowable values for the CFCM concepts. The CFCM runs according to the algorithm described in [11] and when an equilibrium region is reached the CFCM ceases to interact. Then the values of the decision/diagnosis concepts are examined to determine if there is a distinct decision/diagnosis or not. A distinct outcome is inferred, if the value of a decision concept is surpassing the others by at least 10%, in this case the leading competitive node is the suggested decision. Otherwise, when the percent difference between the two leading competitive nodes is less than 10%, then the comparison made in the "Distinct Outputs" box leads to a "NO" result, activating the CBR component. The patient data is then input into the CBR leading to a nearest neighbor search between the patient data and stored cases. Once a case is found with the minimum distance from the patient case, its decision is used to update the CFCM weights and the CFCM is run again until convergence is reached with distinct outcomes, as defined above.

5.2 *Timed Fuzzy Cognitive Maps with Hidden Markov Models*

In Timed Fuzzy Cognitive Maps (T-FCMs), when the difference between the values of decisions concepts is not sufficient to identify a distinct outcome concept, Hidden

Markov Models (HMM) can be used. A HMM represents probability distributions over sequences of observations [4]. A sequence of observations $O = O_1, O_2, \ldots, O_N$ is set to correspond to the concepts-factor values at the time that system has reached to the final state. If the results do not show a clear decision, HMMs are called in order to calculate the probability of the observation sequence given the T-FCM model. Therefore, FCM in synergy with HMM will take action in order to indicate the most probable state for the decision-concept. The synergy with HMM will always reach to a decision based on the most likely state sequence that produced the observation in the model. Using HMM, the system will select the most probable state given the T-FCM model and the sequence of observations. Therefore, this method will lead to select the most probable decision.

6 Applications Areas

Since the introduction of Fuzzy Cognitive Maps there has been an explosion of application domains in which they have been utilized from to modeling of social and behavioral phenomena [19] to process control systems [43] or to medical decision support [15, 16] and many more.

Figure 7 shows a graph of the number of publications per year from 1985 until 2019 (the last complete year prior to this report) using the Scopus database keyword search. It shows that from 1995 there has been a rapid growth in the number of publications. Scopus, has been chosen since it is considered one of the largest abstract and citation database of peer-reviewed literature: scientific journals, books and conference proceedings.

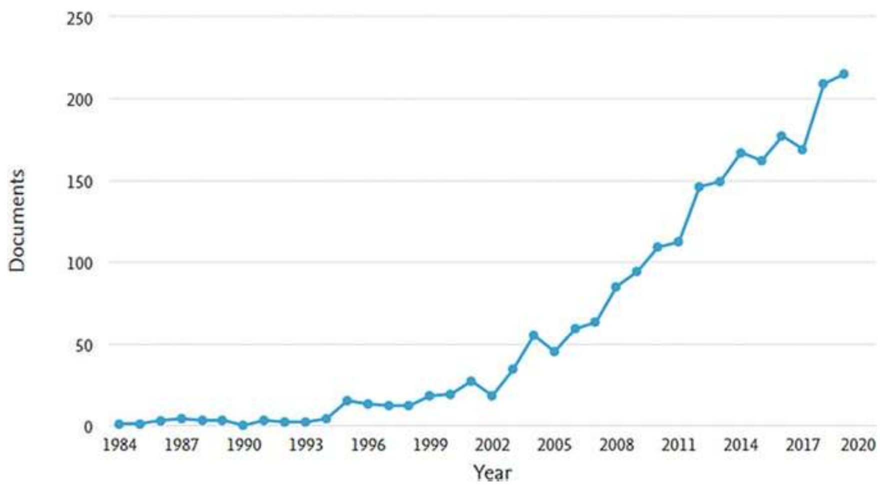


Fig. 7 The exponential growth on FCM publications

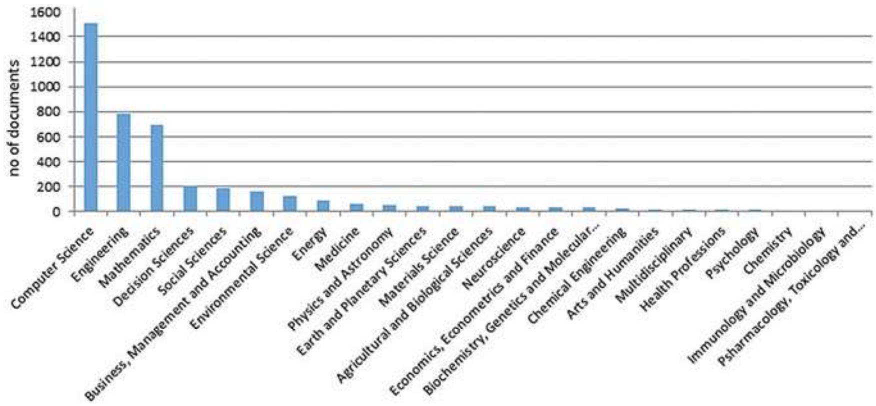


Fig. 8 Number of publications per specific application area based on Scopus database

Figure 8 shows the main application areas based on keywords on published works available in Scopus database. Actually, the general keywords of Computer Science, Mathematics and Engineering have been excluded, even though they dominate the number of publications, since they are, by definition, mathematical, computer science and engineering methodologies. It is clear that FCMs have been applied to solve problems in a wide variety of critical areas including business, social sciences, medicine, agriculture, biology, environment and many more.

7 Main Future Directions

Fuzzy Cognitive Maps have proven to be a significant methodology for causal knowledge acquisition and causal knowledge reasoning. Through synergy of Fuzzy Cognitive Maps with other soft computing, computational intelligence and knowledge-based methodologies various learning and convergence algorithms have been developed making them extremely versatile in their use. This is apparent from the exponential growth of publications and the ever-increasing application areas. Two future directions that are expected to drive the growth of FCMs and applications. The first is derivation of analytical mathematical models of Fuzzy Cognitive Map learning and convergence. This will result in improved system dynamics. The second is the inclusion of time in more FCM architectures will deem FCMs an important tool in modeling of phenomena and processes where time evolution is of key importance.

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