Empower Fuzzy Cognitive Maps Decision Making abilities with Swarm Intelligence Algorithms

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Abstract—The proposed hybrid algorithm aims at defining an FCM by the information carried by a large dataset about a specific problem, taking into consideration the BCO and FPO principles. The link between them is represented by the application of the DB-Scan clustering technique allowing to identify the right number of cluster without knowing it a priori.

The hybridisation highlights the efficacy of the algorithm in estimating the correlations among the factors involved for a specific problem, with low RMSE and computational time, demonstrated by the case study example.

I. INTRODUCTION

In daily life, decisions are continuously made. In some cases, decisions are automatic, but in most cases making a decision is long, demanding, complex and challenging process. Many researchers have focused on studying and developing new decision-making strategies since best choices are affected by the way individuals perceive, represent and process the information involved in the selection problem [1] developing several Soft Computing applications and, in particular, Swarm Intelligent (SI) based algorithms. SI refers to the collective behaviour of decentralised, natural or artificial self-organized systems consisting of a population of simple agents interacting each other and with the environment and following rules [2]. Moreover, relations among agents lead to the emergence of an intelligent global behaviour, unknown to each single agents. On the other hand, soft computing applications have been successfully applied in the DSS area. Fuzzy Cognitive Maps (FCM) represent a soft computing approach able to model systems and support decision-making procedures based on human reasoning approach [3]. According to [4], several problems can benefit from a model based on people’s knowledge and perception about a specific problem and, more particularly, for those in which the stakeholder's opinion is interesting. According to the majority of the soft computing techniques, human knowledge and experience are distinctive in the model through a human-based designing and creation procedure that can be identified in the FCM structure. Different Fuzzy Cognitive Map approaches have been proposed because of the absence of a standardization [5] also integrating SI algorithms. Thus, considering the involvement of several experts, with different expertise, for the problem resolution through FCM, it is possible to consider each expert as a system agent. On these grounds, for example, [6] developed a learning algorithm for Fuzzy Cognitive Maps based on the application of a swarm intelligence algorithm: the Particle Swarm Optimization. Their approach was applied to detect weight matrices leading the Fuzzy Cognitive Map to desired steady states so refining the initial weight provided by the experts and overcoming some deficiencies of other FCM learning algorithms. Always referring to the Particle Swarm Optimisation, [7] to couple desalination technologies and renewable energy systems composed of photovoltaics, batteries and wind turbines. [8] developed an automatic algorithm for the FCM designing according to the Imperialist Competitive Algorithm asserting that it clearly represent a robust, fast and accurate FCM learning algorithm, more than others. Also, [9] referred to the Artificial Bee Colony Optimisation algorithm to develop a FCM model to be applied for an ERP management system. Despite this list is not exhaustive, it highlights the common procedure in modifying the experts’ knowledge with learning algorithms but, according to [10], reducing the human expertise with automated candidates or completely removing their contribute. For this reason, here we present a hybrid algorithm SI based as methodology to model an FCM using large dataset without the experts’ contribution. Specifically, FCM theory is combined with the Flower Pollination Optimisation (FPO) and The Bee-Colony Optimisation algorithms. Some papers have been found in literature about a possible hybridisation of the BCO and the FPO algorithms, but the those realised by [11] and [12] are noteworthy. Specifically, [11] developed a hybrid algorithm refining the FPO solution through the use of BCO to enhance the randomness property of FPO via the simplex method. On the other side, [12] developed the hybridisation referring to the application of the Fuzzy C-means clustering method. However, as already mentioned, the algorithm presented in this study refers to a combination of FPO and BCO considering additionally a third component such as the FCMs for the cost function definition. This the paper is structured as following: Section II describes the theoretical background about the FCMs, the Flower Pollination Optimisation (FPO)
and The Bee-Colony Optimisation algorithms and, more in-depth, the proposed hybrid algorithm. Section III briefly explains an example in order to comprehend the system outcomes, collected and discussed in Section IV. Finally, Section V summarises the study conclusions with suggestions for further expansions and explanations.

II. BACKGROUND

A. The Flower Pollination Optimisation (FPO) algorithm

[13] introduced the Flower Pollination Optimisation (FPO) algorithm paying attention, from the biological evolution point of view, to the plant evolution process. The FPO takes in consideration two natural process: the “global pollination” and the “local pollination” [14]. The first one refers to the cross-pollination or allogamy describing the process during which pollination is due to the pollen of a flower of a different plant or by means, for example, bees and bats. The second one refers to the self-pollination that is the fertilisation of one flower from the pollen of the same flower or different flowers of the same plant. This case often occurs in the case of no reliable pollinator is available. In particular, considering $x_k$ as the $k$-th flower position array and $Z(x_k)$ its objective function identifying the $k$-th food source profitability, for each algorithm iteration it is possible to calculate the best food source value ($z^*$) and its relative position $x^*$. Moreover, if $p$ identifies the pollination type probability boundary and $p_k$ a random probability value for the $k$-th flower, then, the pollination process could be global or local and the position of the $k$-th flower is modified according to (1) or (2).

$$x_k = x_k + L (x^*-x_k)$$

where $L$ is the Lévy flight value [15] used to mimic the insect's movements efficiently and, in some way, to represent the strength of pollination on a flower, and $\alpha$ is random value, ranging in $[0,1]$, and it identifies a local random walk from the $k$-th flower position ($x_k$) and one of its random close flowers, $x_j$. The exploitation of the food source is abandoned when food is no more available or the maximum iterations number ($MaxIt$), defined for the algorithm, is reached.

$$x_k = x_k + \alpha (x_j-x_k)$$

B. The Bee-Colony Optimisation algorithm

[16] introduced the Bee-Colony Optimisation (BCO) algorithm for numerical optimisation taking inspiration by honeybees swarms' behaviour. According to [17], the each bee within the swarm can be classified as “employed”, “scout” or “onlooker”. In the initial phase of BCO, all of the bees are scout and they move within the problem domain space to find a profitable food source. When a food source is related to a specific scout bee, it changes its role in employed bee, starts to harvest pollen and shares its position with the swarm using the so-called waggle dance in the dancing area of the hive. Thus, the onlooker bees can determine the position and the profitability of the food source observing the duration and the glow of the dance [18]. In particular, if $x_k$ is the $k$-th bee position array and $Z(x)$ is the objective function value to be minimised to identify the food source profitability, then, the more profitable is a source, the higher its probability ($p_k$) of selection is, according to (3).

$$p_k = \frac{fit_k x_k}{\sum_{k=1}^{nPop} fit_k x_k}$$

Where $nPop$ is the number of bees belonging to the swarm, and $fit(x_k)$ is the fitness function value related to position of the $k$-th bee ($x_k$) defined to evaluate the food source goodness, according to (4).

$$fit_k(x_k) = \begin{cases} 
\frac{1}{1 + z_k(x_k)}, & z_k(x_k) \geq 0 \\
\frac{1}{1 + [z_k(x_k)]^{z_k(x_k)}}, & z_k(x_k) < 0 
\end{cases}$$

The exploitation of the food source is abandoned when food is no more available or if employed bees’ position cannot be improved within user-specified abandonment criteria. In these situations, the employed bees become scouts and randomly search for new food sources until the maximum iterations number ($MaxIt$), defined for the algorithm, is reached.

C. The Fuzzy Cognitive Map approach

A cognitive map (CM) can be thought as a concept map describing the mental process, gathering information and defining cognitive abstractions, through an individual filter, concerning physical phenomena and experiences [19]. Cognitive maps are visual representations of an individual’s “mental model” constructs, analogous to concept maps for representing human reasoning and knowledge or beliefs.

Thus, considering a generic problem, an experts’ panel is established for an in-depth analysis, since different individuals may face the same problem in a different way, according to their own area of expertise through fuzzy logic, in order to model a collective Fuzzy Cognitive Map (FCM) identifying concepts and relationships about the considered problem. In particular, concepts, in number of $N$, are the FCM key elements that stand for the main characteristics of the abstract mental model for whichever complex system. Once concepts are identified, experts are asked to assign a numerical $w_{ij}$ (the weight of the relation between concept $i$-th and $j$-th) for the $W$ matrix, representing the influence between concepts $C_i$ and $C_j$ according to (5). It ranges in $[-1,1]$. Specifically, $w_{ij} = 0$ indicates no causality relation between concepts; $w_{ij} > 0$ indicates that if $C_i$ increases then, also $C_j$ increases (or $C_j$ decreases and $C_i$ increases), and $w_{ij} < 0$ describes negative causality so if $C_i$ decreases then $C_j$ increases (and viceversa).

$$FCM = \begin{bmatrix} w_{1,1} & \cdots & w_{1,N} \\
\vdots & \ddots & \vdots \\
w_{N,1} & \cdots & w_{N,N} \end{bmatrix}$$

Although several studies exist with respect to the dynamical representation of an FCM, generally, with concern to the aggregation of the experts’ opinions for the collective matrix designing, the provided fuzzy experts’ estimate are gathered using the SUM method [20] and, then, a final linguistic weight is calculated referring to the centre of gravity (COG) defuzzification rule [21]. Some examples are presented by [22]–[25] where a unique credibility value is assigned to each expert and a threshold function is used in the aggregation. On the other hand, a modification of the above mentioned approach has been provided by [26], [27] introducing a corrective factor for an experts’ credibility value if, and only if, his/her judgement is too much dissonant with the others.
This however, does not consider that in a complex multidisciplinary problem experts can have deep knowledge of some parts of the problem.

Once designed the total weights matrix, \( W \), it is possible to identify the system behaviour using simulations. Indeed, if \( A_i \) defines the instantaneous value of \( C_i \), its can be evaluated calculating the influence of the interconnected concepts \( C_j \) on it according to (6).

\[
A_i^{k+1} = f \left( A_i^k + \sum_{j=1}^{n} A_j^k w_{ij} \right)
\]

where \( A_i^{k+1} \) is the value of concept \( C_i \) at simulation step \( k+1 \), \( A_i^k \) is the value of concept \( C_j \) at simulation step \( k \), \( w_{ij} \) is the weight of the interconnection from concept \( C_j \) to concept \( C_i \) and \( f \) is an appropriate threshold function, used to force the concept value to be monotonically mapped into a normalised range [28]. Other equations can be used in place of (5) as suggested by [29].

III. POWERFULL HYBRID ALGORITHM

The proposed hybrid algorithm aims at defining an FCM by the information carried by a large dataset about a specific problem, taking into consideration the BCO and FPO principles.

The bees’ classification is preserved, for as concern BCO, as well the global and local pollination of the FPO. Thus, the link between them is represented by the application of the DB-Scan clustering technique [30]. The choice of DB-Scan lies in its main characteristic: identifying the right number of cluster for a considered population without knowing it a priori. Traditionally, DB-Scan considers a parameter \( \varepsilon \) to specify a distance threshold under which samples are considered to be close, and the minimum number of points belonging to a cluster (nClu). In the proposed algorithm nClu is equal to 1 and \( \varepsilon \) is obtained as median of the distances among the individuals. Clusters composed of 1 individuals are considered outliers.

Through the application of the DB-Scan clustering, fixed the population size equal to nPop such that nPop=nBee+nFlo, with nBee the number of involved bees and nFlo the number of involved flowers, it will be possible identify three kinds of clusters:

- **Flowers Cluster** – since no bees are in this cluster, the flower pollination can only be due to the local pollination and, particularly, from the best flowers belonging to the considered cluster;
- **Bees Clusters** – if the clusters is composed exclusively of bees, no food sources (flowers) are close them so they have to seek a possible suitable food source as a scout bee, within the problem domain space;
- **Mixed Clusters** – if the clusters is composed of bees and flowers, the best bee and the best flower are identified and the global pollination occur;
- **Outliers** – if a bee or a flower does not belong to any cluster, they are classified outlier. Outliers bees are far from flowers and other bees so they became scout bees, outliers flowers are far from flowers and bees highlighting the impossibility of being pollinated so they die.

According to their classification with the clustering procedure, each bee and flower updates its position within the problem space domain, starting from an initial random one in the form of (7).

\[
x_{\text{bee}}(i) = \text{random}((-1; 1), (1, C^2)) \quad i = 1, 2, ..., n\text{bee}
\]

\[
x_{\text{Flo}}(j) = \text{random}((-1; 1), (1, C^2)) \quad j = 1, 2, ..., n\text{Flo}
\]

Where \( x_{\text{bee}}(i) \) and \( x_{\text{Flo}}(j) \) represent the position of the \( i \)-th bee and \( j \)-th flower, respectively. Since the proposed algorithm aims at identifying the best FCM for a specific problem analysis and, since an FCM is composed of values ranging in (-1; 1), also the positions of all the individuals range in the same interval. Moreover, considering the available dataset \( D \) composed of \( S \) samples and described by \( E \) attributes, the numbers of involved concepts (\( C \)), modelling the desired FCM, is equal to the attributes number (\( C = E \)). This means that each positional array is composed of one row and \( C^2 \) columns, \((I, C^2)\).

In light of what has been said, if an individual (bee or flower) is classified outlier during the iteration \( k \) with \( k=1, 2, ..., \text{MaxIt}(\text{MaxIt}) \) is the maximum number of iteration defined by decider for reaching the problem solution), it updates the position according to (7).

If an individual belongs to a Bees Cluster, no food sources (flowers) are close it so it has to seek a possible suitable food source as a scout bee according to (7).

If an individual belongs to a Flowers Cluster, the flower pollination can only be due to the local pollination and, particularly, from the best flowers belonging to the considered cluster according to (8).

\[
x_{\text{Flo}}(j)^{k+1} = x_{\text{Flo}}(j)^k + L \cdot (x_{\text{Flo}}(j)^k - x_{\text{Flo,Best}})
\]

Where \( x_{\text{Flo}}(j)^k \) and \( x_{\text{Flo}}(j)^{k+1} \) are the position of the \( j \)-th flower at iterations \( k \) and \( k+1 \), respectively, and \( x_{\text{Flo,Best}} \) is the best flower in the analysed cluster at iteration \( k \). L is the Lévy flight value [15] used to mimic the insect’s movements efficiently.

If an individual belong to a Mixed Cluster, its position at the iteration \( k+1 \) will be updated with respect to the best individual position into the cluster (bee or flower). According to (9).

\[
x_{\text{type}}(j)^{k+1} = x_{\text{type}}(j)^k + L \cdot (x_{\text{type}}(j)^k - x_{\text{Best}}^k)
\]

Where type represents the possibility that the individual is a bee or a flower (type \( \{\text{Flo, Bee}\} \)) and \( x_{\text{Best}} \) is the best individual position identified among all of the bees and flowers belonging to the cluster at iteration \( k \).

A cost function is necessary to identify the best individual within the swarm population since it allows to calculate the goodness of a food source and so the best solution to the analysed problem.

Once defined a position for each individual and iteration \( (x_{\text{type}}(j)^k \) with \( j=1, 2, ..., n\text{Pop} \) and \( k≤\text{MaxIt} \)) it is possible to calculate the relative cost value according to (6). The
components of the positional array identify the relationships weight of the related FCM, and this implies that, before using (6), the positional arrays have to be converted in matrix form, as described by (10).

\[
FCM^k_j = \begin{bmatrix}
    x^k_j(1) & \cdots & x^k_j(C) \\
    \vdots & \ddots & \vdots \\
    x^k_j(C^2-C+1) & \cdots & x^k_j(C^2)
\end{bmatrix} \quad j = 1, \ldots, nPop
\] (10)

Where \( FCM^k_j \) is the FCM connected with the \( j \)-th individuals at iteration \( k \) and \( x^k_j(m) \) is the \( m \)-th component of the positional array of the \( j \)-th individuals at iteration \( k \) (with \( m=1, 2 \ldots C^2 \)), and it is possible to highlight that \( FCM^k_j \) is equal to (5).

At this point, each sample of the available dataset \( D \) (composed of \( S \) samples and \( C \) attributes) is divided in two parts (according to (6)) to model the initial instantaneous value of FCM concepts \( (A_{i,s,0}) \) and the observed output of the real system \( (x_{i,s,0}) \), as described by (11).

\[
A_{i,s,0} = [D(s, 1) \ D(s, 2) \ \ldots \ D(s, C-1) \ 0] \\
x_{i,s,0} = D(s, C)
\] (11)

At the end of the FCM simulation procedure, the convergence array \( (A_{i,s}) \) is in the form expressed by (12) and \( x_{i,s,C} \) represents the esteemed output.

\[
A_{i,s}^C = \begin{bmatrix}
    a^C_{i,1} & a^C_{i,2} & \cdots & a^C_{i,C-1} & a^C_{i,C}
\end{bmatrix} \\
x_{i,s,C} = a^C_{i,s}
\] (12)

With \( i=1,2 \ldots nPop \) and \( s=1,2 \ldots S \). In particular, the threshold function \( f() \) in (6) is hyperbolic tangent function with \( \lambda=1 \).

When for each individual and for each dataset sample the esteemed value \( x_{i,s,C} \) is stored, the positional cost \( (pC_i) \) is based on the root mean square error formula, expressed by (13).

\[
pC_i = RMSE_i = \sqrt{\frac{1}{nPop} \sum_{j=1}^{S} (x_{i,s,C} - x_{i,s,0})^2} \] (13)

Where \( i=1, 2 \ldots nPop \). Hence, the individual with the lower RMSE value will be considered the "Best". The algorithm functioning is summarised in Fig. 1.

IV. A SHORT CASE STUDY

The case study proposed in this paper aims at identifying an overall indicator in order to rank properly different areas of the Chicago city, with respect to six socioeconomic indicators of public health significance, and help managers in making decisions about investments for the most critical areas.

The considered dataset is available online at [https://data.cityofchicago.org](https://data.cityofchicago.org) (last access: 27/02/2019) and the involved variables take into consideration:

1. **Unemployment (I1)** – defined as the percent of the unemployed population greater than 16 years old;
2. **Dependency (I2)** – the percentage of the population under 18 years old or 64 years old;
3. **Education (I3)** – the percentage of the population over the age of 25 with no high school education;
4. **Income Level (I4)** – the per capita income;
5. **Crowded Housing (I5)** – that is the percent of occupied housing units with more than one person per room;
6. **Poverty (I6)** – the percent of people below the federal poverty level.

As output, a higher *Intercity Hardship Index (O1)* score signifies worse economic conditions. The total number of samples involved in the dataset is equal to 77 (\( S=77 \)).

Thus, the number of concepts involved in the problem, sizing the FCM, is equal to seven (\( C=7 \)). In particular, according to (11), the dataset is manipulated as follow in order to allow the positional cost calculation through (6) and (13).

\[
A_{i,s,0} = [I_1(1) \ I_1(2) \ I_2(3) \ I_2(4) \ I_3(5) \ I_4(6) \ 0] \\
x_{i,s,0} = O_s(7) \quad i = 1, 2, \ldots, nPop \quad s = 1, 2, \ldots, 77
\]

V. RESULTS AND DISCUSSION

In order to test the algorithm efficacy, the available dataset has been divided in two sub-datasets: one for the algorithm training \( (TrD) \) and one for the algorithm testing \( (TeD) \) both with a samples number equal to \( S/2 \) \( (TrD_{sample}=34, TeD_{sample}=33) \) selected randomly within the given dataset.

Moreover, during the training phase of the algorithm, the results of the hybrid one have been compared with those

![Figure 1: the hybrid algorithm flowchart](image-url)
obtained by the application of FPO and BCO. The simulations plan has been realised considering the number of bees (nBee) and flowers (nFlo), and the maximum number of iterations (MaxIt) as setting parameters. Specifically, the possible value has been defined such that nBee, nFlo and MaxIt ranged in [20 40 60 80 100] so, considering all the possible combinations, 125 conditions have been evaluated, and each of them has been simulated by means 100 repetitions because of the use of Lévy flight value. It is important to highlight that, despite the hybrid algorithm discriminates the population members in bees and flowers, FPO and BCO consider the total amount of members (nPop, nPop=nBee+nFlo).

Table I shows an excerpt of the results for the simulations with maximum number of iterations equal 60 and population size variable. It highlights how the Hybrid algorithm outcome provides, for each combination of bees and flowers, the minimum value in terms of RMSE and, in particular, the best one is reached with respect to the combination of 20 bees and 40 flowers (mean value over 100 repetitions equal to 0.17 and standard deviation equal to 0.05).

<table>
<thead>
<tr>
<th>nBee</th>
<th>nFlo</th>
<th>nPop</th>
<th>Hybrid</th>
<th>BCO</th>
<th>FPO</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>20</td>
<td>40</td>
<td>0.31</td>
<td>0.92</td>
<td>0.53</td>
</tr>
<tr>
<td>40</td>
<td>20</td>
<td>60</td>
<td>0.24</td>
<td>0.65</td>
<td>0.69</td>
</tr>
<tr>
<td>60</td>
<td>20</td>
<td>80</td>
<td>0.2</td>
<td>0.85</td>
<td>0.51</td>
</tr>
<tr>
<td>100</td>
<td>20</td>
<td>120</td>
<td>0.28</td>
<td>0.68</td>
<td>0.68</td>
</tr>
<tr>
<td>20</td>
<td>40</td>
<td>60</td>
<td>0.17</td>
<td>0.6</td>
<td>0.63</td>
</tr>
<tr>
<td>40</td>
<td>40</td>
<td>80</td>
<td>0.24</td>
<td>0.68</td>
<td>0.8</td>
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<tr>
<td>60</td>
<td>40</td>
<td>100</td>
<td>0.34</td>
<td>0.83</td>
<td>0.74</td>
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<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>40</td>
<td>80</td>
<td>120</td>
<td>0.62</td>
<td>0.87</td>
<td>0.47</td>
</tr>
<tr>
<td>60</td>
<td>80</td>
<td>140</td>
<td>0.21</td>
<td>0.84</td>
<td>0.52</td>
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<td>...</td>
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<td>...</td>
</tr>
<tr>
<td>80</td>
<td>100</td>
<td>180</td>
<td>0.25</td>
<td>0.79</td>
<td>0.6</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>200</td>
<td>0.22</td>
<td>0.5</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Conversely, BCO and FPO reach the minimum RMSE considering, respectively, a total population size (nPop) equal to 200 (mean value equal to 0.5 and standard deviation equal to 0.12) and 120 (mean value equal to 0.47 and standard deviation equal to 0.13). This means that the hybrid method reaches the best solution in less time and with a minor computational effort. At the same time, it is possible to underline that the hybrid algorithm reaches better results when the number of flowers is greater than the number of bees, as explained, for example, by comparing the two lines highlighted in grey.

Considering all the 125 simulated conditions over 100 repetitions, the hybrid algorithm RMSE has mean value equal to 0.27 (standard deviation equal to 0.08) whilst BCO by a mean value equal to 0.77 (standard deviation equal to 0.23) and FPO by mean value equal to 0.71 (standard deviation equal to 0.20). This observation can demonstrate how the hybridisation proposed provides better results in the FCM designing. For as concern the best solution within the whole simulation plan, the hybrid algorithm has RMSE value equal to 0.15 with standard deviation equal to 0.051 in relation with 20 bees, 40 flowers and 80 iterations and the esteemed FCM is described in Table 2. Fig.2 shows the comparison among the outcomes of the hybrid algorithm and BCO and FPO ones. It highlights how BCO (dashed line) is little conditioned by the number of iterations since its behaviour is very stable and reach the convergence in very little time. Conversely, FPO (dotted line) increases its accuracy depending on the number of the iterations but this means that a good solution is reached only with a high number of iterations. However, both BCO and FPO find their solution with a high value of RMSE. On the other hand, at same conditions, the hybrid algorithm reaches a better solution independently to the number of iterations with a trend more stable and with a high convergence velocity.

<table>
<thead>
<tr>
<th></th>
<th>I(1)</th>
<th>I(2)</th>
<th>I(3)</th>
<th>I(4)</th>
<th>I(5)</th>
<th>I(6)</th>
<th>O(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I(1)</td>
<td>0.00</td>
<td>0.58</td>
<td>-0.19</td>
<td>-0.60</td>
<td>-0.26</td>
<td>-0.26</td>
<td>0.10</td>
</tr>
<tr>
<td>I(2)</td>
<td>-0.08</td>
<td>0.00</td>
<td>-0.22</td>
<td>-0.04</td>
<td>0.50</td>
<td>-0.04</td>
<td>0.44</td>
</tr>
<tr>
<td>I(3)</td>
<td>-0.17</td>
<td>-0.25</td>
<td>0.00</td>
<td>-0.10</td>
<td>-0.20</td>
<td>-0.37</td>
<td>0.31</td>
</tr>
<tr>
<td>I(4)</td>
<td>-0.03</td>
<td>0.20</td>
<td>0.51</td>
<td>0.00</td>
<td>-0.80</td>
<td>0.34</td>
<td>0.73</td>
</tr>
<tr>
<td>I(5)</td>
<td>-0.51</td>
<td>0.90</td>
<td>-0.07</td>
<td>0.51</td>
<td>0.00</td>
<td>0.27</td>
<td>-0.13</td>
</tr>
<tr>
<td>I(6)</td>
<td>0.10</td>
<td>0.56</td>
<td>0.18</td>
<td>-0.04</td>
<td>-0.57</td>
<td>0.00</td>
<td>-0.77</td>
</tr>
<tr>
<td>O(1)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Once verified the algorithm accuracy and obtained the esteemed FCM, it has been used on the Testing Dataset to calculate the Intercity Hardship Index to help mangers in making decisions about the best investments plan. An excerpt of the results is reported in Table 3 and highlight a mean estimation error equal to 0.03 with standard deviation equal to 0.02.

VI. CONCLUSION

The proposed paper presents a hybrid algorithm SI based as methodology to model an FCM using large dataset without the experts’ contribution avoiding all the problems and criticalities due to the use of experts’ knowledge. The hybridisation of BCO and FPO, jointly to the application of FCM theory, as described by the discussed results, highlights the efficacy of the algorithm in estimating the correlations.
among the factors involved for a specific problem, with low RMSE and computational time.

### TABLE 3: ESTEEMED OUTPUT BY THE TESTING DATASET

| Sample | $x_{i,s,C}$ | $x_{i,s,O}$ | $|x_{i,s,C} - x_{i,s,O}|$ |
|--------|-------------|-------------|-----------------|
| 1      | 0.644       | 0.630       | 0.014           |
| 2      | -0.754      | -0.755      | 0.001           |
| 3      | 0.153       | 0.068       | 0.084           |
| 4      | 0.017       | 0.012       | 0.005           |
| 5      | 0.104       | 0.106       | 0.002           |
| ...    | ...         | ...         | ...             |
| 29     | 0.712       | 0.742       | 0.029           |
| 30     | 0.185       | 0.218       | 0.033           |
| 31     | -0.402      | -0.325      | 0.077           |
| 32     | 0.335       | 0.368       | 0.032           |
| ...    | ...         | ...         | ...             |
| 36     | -0.474      | -0.456      | 0.018           |
| 37     | -0.581      | -0.568      | 0.013           |
| 38     | -0.661      | -0.661      | 0.000           |

For further researches, the application of the algorithm in a real case scenario and its comparison with traditional outcomes are compulsory in order to validate itself efficiently.

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### REFERENCES


