Timed Fuzzy Cognitive Maps

Evangelia Bourgani
Dept. of Computer Science & Engineering,
University of Ioannina,
Ioannina, Greece
ebourgani@gmail.com

Chrysostomos D. Stylios
Dept. of Computer Engineering, Technological Educational Institute of Epirus,
Artas, Greece
stylios@teiep.gr

George Manis
Dept. of Computer Science & Engineering,
University of Ioannina,
Ioannina, Greece
manis@cs.uoi.gr

Voula Georgopoulos
School of Health and Welfare Professions,
Technological Educational Institute of Western Greece,
Patras, Greece
voula@teipat.gr

***Abstract*—Taking time into consideration in the FCM model and the subsequent numerical calculation is a hot FCM issue. This work investigates an approach for introducing the time in Fuzzy Cognitive Maps (FCM) framework and it proposes the Timed-FCM (T-FCM) that incorporates time steps in their structure and constitutes an extension of FCMs. The proposed T-FCMs take into consideration the time evolution and the fact that the parameters of any system change with the time and it accepts intermediate states and results of the modeled system. In any problem, its factors change differently over time and so their influence as is presented on the interconnections between the concepts are time dependent.**

Keywords—Fuzzy Cognitive Map, Soft computing, modeling, time evolution

I. INTRODUCTION

Most of today's models and systems are characterized by high complexity but at the same time the requirements for optimal performance are continuously higher. This is supported by the available computational power that permits to propose and adopt approaches that require a lot of storage and/or computations provided that will lead to better models with great performance.

Any real world problem comprises time as either the system itself or the external inputs change over time and so its outputs are time dependent. Any successful modeling approach has to take into consideration time so that to best describe the operation of any system. The time is fundamental, it determines the behavior of a system, all the elements of a model change over time.

Fuzzy Cognitive Maps (FCM) have been applied to model systems and especially to develop decision support systems for discipline application areas. One significant advantage of FCM modeling approach is the fact that FCMS can handle vague, even incomplete or conflicting information. Such an advantage is essential especially for cases where many factors should be included in the model and have to be taken under consideration before reaching any decision. The FCM modeling approach is symbolic, presenting abstract knowledge and is based on human expert experience and knowledge. FCMs have been used successfully to develop Decision Support Systems (DSS) because of their ability to involve all the main factors influencing any decision. On the other hand, factors are frequently time dependent and this work investigates the introduction of time in the FCM model and how it would be described, developed and operated in a FCM model that incorporates time. It is expected that ntroduction of time into FCM model will provide better solutions to real world problems.

Section 2 of this study describes the classical FCM structure, the distinction between synchronous and asynchronous FCMs and a short review on research efforts to introduce time at FCMs. Section 3 presents an approach that incorporates time into the FCM structure and it explains how the T-FCM is developed. Section 4 compares T-FCMs vs FCMs and discusses the possible range of applications of Timed FCMs. Section 5 concludes the paper, it provides future directions and challenges.

II. FUZZY COGNITIVE MAP

A. The structure

Fuzzy Cognitive Map (FCM) is a soft computing modeling technique that has applied to model complex systems. FCMs were introduced [1] as an extension to Cognitive Maps [2] by including fuzzy logic ideas. FCMs are able to represent causal knowledge they supports cause- effect reasoning process. FCMs belong to the soft-computing systems that can solve decision making problems, modeling and control problems.

FCMs resemble the human reasoning approach; they rely on the human expert knowledge for a domain, they make similar associations along generalized relationships among the domain descriptors i.e. concepts and so they infer suggestion and conclusions. FCMs can model any real world system as a collection of interacting concepts and cause- effect relations among concepts.

FCM is an illustrative causative representation for the description and modeling of any system. The graphical representation consists of interconnected concepts. FCMs are dynamical, fuzzy signed, directed graphs, promoting feedback. Where the weighted edge w_{ij} from causal concept C_i to affected concept C_j describes the degree by which the first concept influences the latter, as Fig.1 illustrates.

Experts design and develop the structure of the system, they determine the nodes that represent the key factors of the system operation. They describe interconnections between concepts using linguistic variables to correspond to the cause effect relationships among concepts. Then all the linguistic weight variables are combined and an aggregated weight is assigned to every interconnection.

The concepts of the FCM correspond to main inputs, outputs, variables, states, factors and other main characteristics that could be used to model and describe the operational behavior of the system.

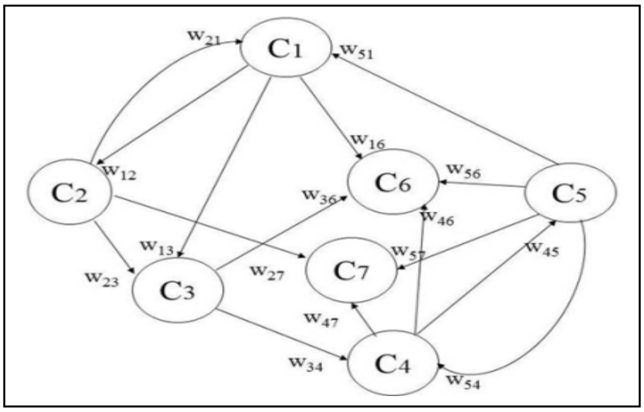

Fig. 1. The FCM model

FCM is a conceptual network, which is in most of the cases built by experts, using an interactive procedure of knowledge acquisition. The connections among the signed and weighted arcs represents the causal relationships. The sign of the weight shows the kind of causal relationship between concepts. That is, if $w_{ji}>0$ then any increase in the value of concept C_j cause increase in the value of concept C_i; if the weight is $w_{ji}<0$ then when value of concept C_j increases this has an inverse influence and value of C_i is decreasing and vice versa. While if $w_{ji}=0$ concept C_j has no causal effect on C_i. The mathematical model of an FCM is described with eq.1.

$$A_i^{k+1} = f(A_i^k + \sum_{\substack{j \neq i \\ j=1}}^{N} A_j^k w_{ji}), \tag{1}$$

where A_i^{k+1} is the value of concept Ci at simulation step k+1, A_j^k the value of the interconnected concept C_j at step k, w_{ji} is the weight of the interconnection between concept Cj and Ci, and f is a sigmoid threshold function (2):

$$f(x) = \frac{1}{1+e^{-\lambda x}} \tag{2}$$

Where $\lambda>0$ is a parameter determining its steepness.

B. Synchronous and Asynchronous FCM

There are two main approaches characterizing the way that Fuzzy Cognitive Maps interact either in a synchronous process or in an asynchronous one. This main distinction refers to the time scale of the influence from one concept to the interconnected ones. It is reflected on the calculation approach, which is used to update the values of concepts and it described how FCM is running.

In the case of synchronous FCMs, all the values of concepts are updated concurrently using equation 1. But for the case of asynchronous FCMs, the interaction among concepts takes place in different time scales.

Another distinction is referred to the way that the weight among concepts is updated synchronously or asynchronously. For the asynchronous weight updating, there proposed learning algorithms [3] that proved that FCM outperforms synchronous weight updating [4]. This is justified because asynchronous weight updating create advanced FCMs with dynamic behaviour and great modelling abilities, where new features can easily be introduced, added or deleted allowing a model to continuously evolve.

C. Existing attempts to introduce time on FCMs

There are some efforts to insert time dimension into the FCM framework. Time is a significant parameter for any model that is used to describe and simulate real problems; however it is quite difficult to incorporate it into a process during runtime.

In 1992, Hagiwara proposed the extended FCM (e-FCM) [5]. He briefly described and referred to the need for introducing nonlinear weights, conditional weights and time delays weights in the FCMs. Park introduced the Fuzzy Time Cognitive Maps (FTCMs) by inserting dummy nodes used to show and achieve time lags between nodes [6]. Carvalho *et al.* proposed Rule Based FCM (RB-FCM); they used a parameter called B-Time representing the highest level of temporal detail that a simulation can provide in the modelled system [7]. Miao *et al* introduced dynamic functions for the arcs to represent the dynamic and temporal effects of causal relationships [8]. Zhong *et al.* proposed the Temporal FCMs. Temporal FCM (tFCM) and they introduced a discrete linear temporal domain, where a detailed model is described and each time concept value is determined by various effects [9].

All these attempts aimed to improve FCMs and to make them more dynamic, more efficient and more suitable for real life models. However, they have not been widely adopted as other used just for numerical weights, other used with fixed time lags or a unique B-Time, while others were highly complicated.

III. TIMED FCM STRUCTURE

A. *Introducing Timed Fuzzy Cognitive Map*

As it has been explained above, all real world problems incorporate time; actually time scale is embedded in any system. If we want to create an advanced model, which will be able to provide solutions, it has to include the meaning of evolution in time.

Here an approach is proposed which incorporates time into the system in a dynamic and transparent way, so that the user will be able to interpret within the system. The proposed approach relies on experts, literature and any other source that could contribute valuable information. Fig 2 shows the main steps to construct the Timed FCMs and these are calculated:

Construction procedure

Step 1: Determination of all C_{CI} to C_n concepts (using experts and information from literature) [11]

Step 2: For all possible time units suggest

$$\min\{t_1,t_2,..t_n\},$$

Step 3: Introduced of the combination of $\gamma_{1,...}\gamma_n$. Where γ correspond to the possible additional parameters that have to be incorporated in the model. Generally, it is γ= [0,1] so that it defines a unique situation

Step 4: For every t and for the corresponding interrelationship, a set of weights are defined for the interconnection $C_i \rightarrow C_j$, notated as $w_{\gamma,t,ij}$

If there are available *k* experts, a credibility weight cr_k has to be assigned to every expert, which is used in the calculation: $cr_k * w_{\gamma,t,ij}$

Step 5: For a case γ and for t=0→ t=t, the interrelationships from one concept to another will be:

$$d^{t+1}_{\gamma,t,i,j} = f(d^{t}_{\gamma,t,i,j} + \sum_{\substack{j\neq i \\ j=0}}^{t-1} d^{t-1}_{\gamma,t,i,j}) = f(w,\gamma,t,C_i,C_j)$$

Step 6: Iterate step 5 until it will FCM will convergence i.e.

If ($d^{t}_{\gamma,t,i,,j}$ < an acceptable value) {a decision is reached }

else { redesign is demanded}

The above algorithm is described and consists of the following steps:

Concepts: In this step the concepts which constitute the FCM are determined. If we are modeling a decision support system, concepts are divided in decision and factor concepts. Factor concepts include those elements that can be affected by each other and they can influence the decision concepts so that through interconnections to conclude to a decision. Decision concepts describe the different suggestions of the possible solution. An expert determines the main concept based on his experience, knowledge and observations on system operation. A further step would be to insert a different credibility weight to each one of the concepts that come from certain and proven sources, such as literature compared to concepts which are derive from experts' observance. This could be applied if concepts are grouped and evaluated accordingly. But this is beyond the current research and it requires further description and investigation.

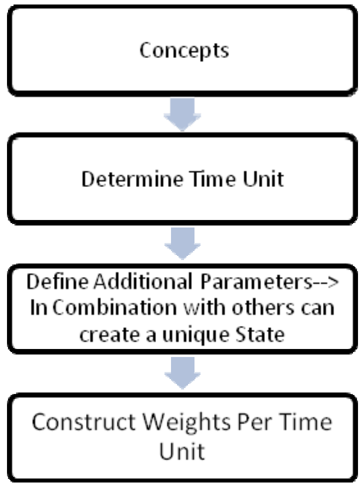

Fig. 2. Basic steps for TFCM construction

Time unit is an important element that experts should define. The procedure to define time unit is essential and depends on the field and the goal of the under investigation issue. Experts have to determine the different time scales when there is long-term interaction among concepts and when there is a shorter time unit among concepts. The reference time unit will be the shorter one, where an important change can affect and provoke a change to the concept. The reference time unit is determined based on the expert's experience and knowledge, as they have faced similar real problems.

Additional parameters. The proposed procedure introduces the so called additional parameters. As additional parameters are characterized the external elements that influence independently each one of the concepts. Based on these parameters there are defined the initial state and the triggering evolution of the overall procedure. Any specific combination of these parameters correspond to a specific case, with its ones initial values. It is supposed that there is developed a Case Base using both experts and recorded cased and for each one case the additional parameters are identified. The additional parameters and their direction of interaction during the time describe the behaviour of the overall procedure. Thus the values of concepts along with the values of the additional parameters correspond to a specific case. If the FCM model is used for a medical problem, the additional parameters will correspond to a patient's specific characteristics..

Table 1 gathers all the possible states that parameters could take. As it has explained any case γ of the case base correspond to specific characteristics. The index n of γ refers to a specific case and from Table I are selected the combination's weights that would be used during the runtime procedure..

TABLE I. DATA BASE WITH ALL THE AVAILABLE ACTIVATED PARAMETERS

γ_n	*Activated parameters (p_1,p_2,..,p_n)*
γ_0	000….0
γ_1	100….0
γ_2	010….0
γ_3	110….0
$\gamma_{...}$	111.....1

Weights per time unit: The initial weights, $w_{\gamma=0,t=0,ij}$ are defined by experts using known procedure [11]. But values of weights may change at every time unit. The weights are described linguistically for each interconnection and for each time unit either by experts or by exploiting the conclusions of related literature, studies and/or other sources. Thus for each interconnection, when time is taken into consideration, the weight of the interconnection takes values proportionally to the time unit. It is mentioned that for each time unit a different weight is assigned. The weight assignment procedure is repeated until all the possible combinations are defined for each additional parameter. In that way all the cases that are gathered at the Case Base are taken into consideration. For any specific case a corresponding weight is calculated of the interconnection from cause concept C_i to effect concept C_j (under the specific activated parameters and time unit), which depends both on the simulated time unit and the state (additional parameters).

Generally, the weights would have the following dependence:

$$\begin{aligned} & w_{\gamma_1,t_1,i,j}, w_{\gamma_1,t_2,i,j} \ldots w_{\gamma_1,t_n,i,j}, \\ & w_{\gamma_2,t_1,i,j}, w_{\gamma_1,t_2,i,j} \ldots. w_{\gamma_2,t_n,i,j} \\ & \ldots. \\ & w_{\gamma_n,t_1,i,j}, w_{\gamma_n,t_2,i,j} \ldots. w_{\gamma_n,t_n,i,j} \end{aligned} \tag{3}$$

Where γ shows the activated state of additional parameters and t_1,t_2… are the time step. Thus, $w_{\gamma,t,ij}$ defines the weight between two concepts C_i and C_j, which is selected according to the activated parameters (γ) for each time unit.

B. *Defining the weights*

The weight definition procedure is a complex one as it has to calculate the weights for every case. Experts recall the progression of the corresponding phenomenon during the time and they suggest the weight for each interconnection and each time unit. Experts define the initial weights, denoting the concepts' dependencies (weights) and they determine the higher or lower influence during the progression of the phenomenon. Additional parameters determine the direction of changes. That is, during the time the interdependences among some factors may have different degrees of influence compared to others and this change depends on both time and additional parameters; their interconnections can become weaker or stronger, while some may be deactivated and other activated.

Experts determine the interconnections using fuzzy rules [10]; they infer a linguistic variable that describes the relationship between the two concepts [11]. That is, after the collecting and definition of concepts, experts are asked to propose the degree of influence among all the concepts, using IF- THEN rules. Their answers will be influenced by their knowledge, experience, research results etc [12]. These rules are according to the statement:

For time unit 0….N:

"IF a *{none, very-very low, very low, low, negatively medium, positively medium, high, very high, and very very high}* change occurs in concept value C_i

THEN a *{none, very-very low, very low, low, negatively medium, positively medium, high, very high, and very very high}* change of value C_j is caused

THUS the influence of C_i to C_j is a TC_{ij} *{influence}*".

In this way, for each interconnection every expert determines the influence with a linguistic weight. All the inferred linguistic weights for each interconnection are aggregated and deffuzzified producing an initial numerical weight in the interval [-1, 1].

However, here the additional parameter have introduced that are different for each one case. In order to model the system more accurately, experts have to take into account these changes that happen during the simulation process, and so to define the change for each time unit. Thus, for each interconnection when there is different time scale along with the presence of additional parameters, new values on each additional parameter would lead to the appropriate weight change. Linguistic degrees are used along with the indication of direction of change; new values such as *none, very-very low, very low, low, negatively medium, positively medium, high, very high, and very very high.*

These degrees correspond to negative or positive direction and the IF THEN statement that experts use are:

For time unit 0 ....N:

"IF *(Additional Parameter p_1 is TRUE (AND/OR Additional Parameter p_2=FALSE, p_3 =TRUE)* AND IF a *{ none, very-very low, very low, low, negatively medium, positively medium, high, very high, and very very high}* change in concept value C_i occurs

THEN a *{ none, very-very low, very low, low, negatively medium, positively medium, high, very high, and very very high}* change in value C_j is caused

THUS the influence of C_i to C_j is a TC'_{ij} *{influence}*

ELSE {keep the default weight of the correspondent time and state (TC_{ij}{influence}).

Thus, the weight for each concept interconnection and for a specific combination of activated concepts, will be given by:

$$w^{t}_{\gamma,t,i,j} = \alpha * w^{t-1}_{\gamma,t,i,j} \quad (4)$$

where α is the coefficient that includes the speed of change between each interconnection. The α is multiple to time t and can take discrete values in [1,2t,.. nt]. The '1' correspond to the case that no change is taking place.

The inferred new fuzzy weights, for each time unit and for each parameter, aggregated and defuzzified giving new numerical weight (in the interval [-1,1]) to the correspondent interconnections. This procedure can approximate better real life situations, as the weights change per time unit incorporating more elements and parameters, taking into consideration every possible situation, no matter how rare or underestimated may be.

If we include all the above information into one table for each state (γ), the interconnections among concepts for each time unit are defined. This information will be contained to the variable $d_{\gamma,t,ij}$, where γ and t will determine the case and the time unit, correspondingly.

Table II presents the weight dependencies ($d_{\gamma,t,ij}$) for different cases. Particularly, $d_{\gamma,t,ij}$ is an array that its dimensions correspond to the number of variables γ and t. Each cell of this array will be a matrix and the dimensions of this matrix will be defined as follows: if the number of concepts is m, then the $d_{\gamma,t,ij}$ will be a $m \times m$ matrix that will contain all the weights among the concepts, for the correspondent time unit and state. Table III contains the values of $d_{\gamma,t,ij}$ for a specific time unit (t1).

TABLE II. INTERCONNECTION MATRIX FOR EACH TIME UNIT AND STATE

γ_n /time unit	γ_0	γ_1	...	γ_n
t_1	$d_{\gamma0,t1,ij}$	$d_{\gamma1,t1,,ij}$	$d_{\gamma\ldots,t1,ij}$	$d_{\gamma n,t1,ij}$
t_2	$d_{\gamma0,t2,ij}$	$d_{\gamma1,t2,ij}$	$d_{\gamma\ldots,t2,ij}$	$d_{\gamma n,t2,ij}$
....	$d_{\gamma0,t\ldots,ij}$	$d_{\gamma1,t\ldots,ij}$	$d_{\gamma\ldots,t..,ij}$	$d_{\gamma n,t\ldots,ij}$

TABLE III. WEIGHTS FOR EACH INTERCONNECTION AND TIME UNIT

	C_1	C_2	.....	C_n
C_1	$w_{\gamma1,t1,11}$	$w_{\gamma1,t1,12}$	.....	$w_{\gamma1,t1,1n}$
C_2	$w_{\gamma1,t1,21}$	$w_{\gamma1,t1,22}$	.....	$w_{\gamma1,t1,2n}$
....	...	...	....	...
C_n	$w_{\gamma1,t1,n1}$	$w_{\gamma1,t1,n2}$	.....	$w_{\gamma1,t1,nn}$

Figure3 illustrates the dependence of the system on the external additional parameters, which activate with the appropriate values the simulation. C_1 to C_4 are the factor concepts, while C_5 and C_6 are the decision concepts, p_1 to p_4 are the additional parameters which their combined value define the unique case (γ) for a time unit. The illustrated example represents a small timed-FCM system with few concepts and few additional parameters for simplicity. Real systems are bigger and more complex.

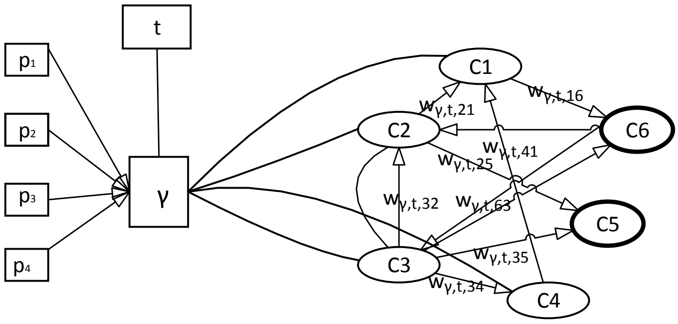

Fig. 3. An example of the dependencies for a small size of FCM

IV. DISCUSSION ON TIMED- FCM

A. *Timed-FCM versus FCM*

Conventional FCM do not describe adequate temporal concepts. However, almost every real problem depends on time. T-FCM introduces the time into FCM model, it describes how time affectes the concept values, their interconnection weights and their evolution over time. Timed FCM lead to a more dynamic system in which the user determines the external characteristics according to the under investigation case and user can intervene on the system by changing concept values in discrete time units. T-FCM concludes to result for every time unit, giving an overview of the case evolution and rendering the user able to judge the result. Thus, the acceptance or not of the final decision will be based on the evolution of the case.

B. *Possible applications of Timed-FCM*

Timed-FCM can be used in every field that time is an essential factor. Most appropriate applications include medicine, economics, environment and so on areas where problems are highly dependent on time. For such cases we will be able to approximate better the real world evolution of different problems, as T-FCM attempt to follow the natural evolution of cases. Therefore, the lack of the concept of time, regarding the order and the reaction time of a change, may provoke important changes in patient state or influence the output in a strategic/economic problem or an environmental problem.

In many medical applications, time has not taken into consideration [13] despite the fact that time is a crucial factor which can lead to different decision. T-FCMs are essential for problems of differential diagnosis [14], [15]} or in obstetrics [16], where time plays a significant role for the evolution of a problem. In such cases, models without temporal concept may lead to the opposite decision.

V. CONCLUSION

In this work we introduced the time into FCM structure. Time is basic for many problems in various fields, as it can influence the interconnections and change the decision. This method tries to incorporate the time by determining, in each time step, the dependence of time among the concepts and quantify this dependence. The proposed method provides a transparent and dynamic system that can change by the user each time step. The initial state (case) is determined by the user who set the external parameters of the problem according to his/her expereience for the under investigation case.

T-FCMs follow the real evolution of any problem. They are developed by taking into account literature, experts' experience, and any other source that could be regarded as reliable.

In future work, we will define and develop the overall procedure of extracting the result/decision and we will implement it into various application problems. T-FCM, however, is also highly dependent on human (expert), and so there is proposed the introduction of credibility weights.

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