Mathematical Formulation of Fuzzy Cognitive Maps*

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Abstract

This paper presents an overview in existing representations of Fuzzy Cognitive Maps (FCM) and a new approach in the formulation of Fuzzy Cognitive Maps is examined. The description and construction of Fuzzy Cognitive Maps (FCM) is briefly represented and some new ideas for the modeling of Fuzzy Cognitive Maps are presented. Research in this area was mainly focalized on the representation, construction and application of FCM, and now in this paper different types and mathematical description of Fuzzy Cognitive Maps are examined and FCMs are mathematically transformed in forms that are analogous to Recurrent Neural Networks. This similarity stimulates the investigation of Forward Accessibility for discrete-time FCM models. Finally, an example of a process is presented and it is formulated in form that controllability aspects can be examined.

1. Introduction

Fuzzy Cognitive Map (FCM) methodology is a symbolic representation for the description and modeling of complex system. Fuzzy Cognitive Maps describe different aspects in the behavior of a complex system in terms of concepts; each concept represents a state or a characteristic of the system and these concepts interact with each other showing the dynamics of the system. FCMs illustrate the whole system by a graph showing the cause and effect along concepts, and are a simple way to describe the system's model and behavior in a symbolic manner, exploiting the accumulated knowledge of the system. A Fuzzy Cognitive Map integrates the accumulated experience and knowledge on the operation of the system, as a result of the method by which it is constructed, i.e., using human experts that know the operation of system and its behavior in different circumstances. Moreover, Fuzzy Cognitive Map utilizes learning techniques, which have implemented in Neural Network Theory, in order to train Fuzzy Cognitive Map and choose appropriate weights for its interconnections.

At first, a political scientist Axelrod (1976) introduced cognitive maps for representing social scientific knowledge and describing the methods that are used for decision making in social and political systems. Then Kosko (1986, 1992) enhanced the power of cognitive maps considering fuzzy values for the concepts of the cognitive map and fuzzy degrees of interrelationships between concepts. After this pioneering work, Fuzzy Cognitive Maps attracted the attention of scientists from many fields and have been used in a variety of different scientific problems.

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2. Different Representation of Fuzzy Cognitive Maps

2.1.1 Fuzzy Cognitive Maps of Type I

Fuzzy Cognitive Maps are fuzzy signed graphs with feedback (Stylios *et al.*, 1997a). They consist of nodes-concepts C_i and interconnections e_{ij} between concept C_i and concept C_j. A Fuzzy Cognitive Map models a dynamic complex system as a collection of concepts and cause and effect relations between concepts. A simple illustrative picture of a Fuzzy Cognitive Map is depicted in Figure 1, consisted of five nodes-concepts.

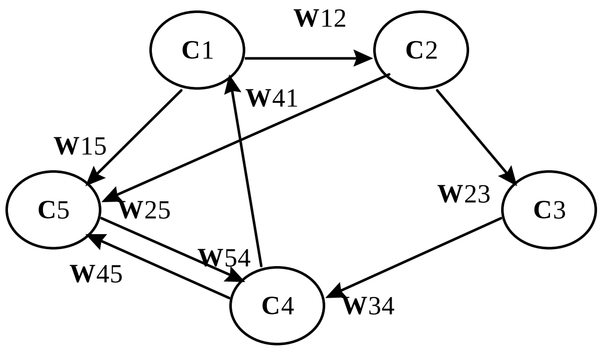

Figure 1. A simple Fuzzy Cognitive Map

Interconnections e_{ij} among concepts are characterized by a weight w_{ij} that describes the grade of causality between two concepts. Weights take values in the interval [-1,1]. The sign of the weight indicates positive causality $w_{ij} \succ 0$ between concept C_i and concept C_j, which means that an increase of the value of concept C_i will cause an increase in the value of concept C_j and a decrease of the value of concept C_i will cause a decrease in the value of concept C_j. When there is negative causality between two concepts, then $w_{ij} \prec 0$; the increase in the first concept means the decrease in the value of the second concept and the decrease of concept C_i causes the increase in value of C_j. When there is no relationship between concepts, then $w_{ij} = 0$. The strength of the weight w_{ij} indicates the degree of influence between concept C_i and concept C_j.

Generally, the value of each concept is calculated, computing the influence of other concepts to the specific concept, by applying the calculation rule of equation (1):

$$x_i(t) = f\left(\sum_{\substack{j=1 \\ j \neq i}}^{n} x_j(t-1) w_{ji}\right) \tag{1}$$

Where $x_i(t)$ is the value of concept C_i at time t, $x_j(t-1)$ is the value of concept C_j at time $t-1$, w_{ji} is the weight of the interconnection between concept C_j and concept C_i and f is the sigmoid function: $f = \dfrac{1}{1+e^{-\lambda x}}$

Other proposed squeezing functions are the $\tanh(x)$, $\tanh(x/2)$ and others that will convert the result of the multiplication into the fuzzy interval [0,1] or [-1,1], where concepts can take values.

At each time step, values for all concepts of Fuzzy Cognitive Map change and recalculate according to equation (1). The calculation rule for each simulation step of FCM includes calculation of new values for all the concepts. It is consisted of a $nx1$ vector $\mathbf{X}$ which gathers the values of n concepts, and the matrix $\mathbf{W} = [w_{ij}]_{1 \le i,j \le n}$ which gathers the values of the causal edge weights for the Fuzzy Cognitive Map, where the dimension of the matrices is equal to the number n of the distinct concepts, which consist the map. So the new state vector $\mathbf{X}$ of the FCM at time t is calculated according to the equation:

$$\mathbf{X}(t)= f(\mathbf{W}^{\mathsf{T}}\, \mathbf{X}(t-1)) \tag{2}$$

2.1.2 Construction and learning for FCM I

It must be mentioned that the use of experts is very critical in the designing and development of Fuzzy Cognitive Maps. Experts who have knowledge and experience on the operation and behavior of the system are involved in the determination of concepts, interconnections and assigning casual fuzzy weights to the interconnections (Kosko, 1992; Stylios and Groumpos, 1999)

Generally, Fuzzy Cognitive Maps can be trained, using learning algorithms in a similar way as in neural networks theory. Proposed learning algorithms belong to the unsupervised learning algorithms. During the training period of FCM, the weights of the map change with a first-order learning law that is based on the correlation or differential Hebbian learning law:

$$w'_{ij} = -w_{ij} + x'_i x'_j \tag{3}$$

So $x'_i x'_j \succ 0$ if values of concepts C_i and C_j move in the same direction, and $x'_i x'_j \prec 0$ if values of concepts C_i and C_j move in opposite directions; therefore, concepts which tend to be positive or negative at the same time will have strong positive weights, while those that tend to be opposite will have strong negative weights.

2.2 FCMs of Type II

A new calculation rule of Fuzzy Cognitive Maps is proposed, which take into consideration the previous value of each concept. Fuzzy Cognitive Map will have one time step memory capabilities, the last value of each concept is involved in the determination in the new value of concept and so the values of concepts will have a slight variance after each simulation step. Here, in order to take into account these observations, a new formulation is presented. Namely, we propose the following equation:

$$x_i(t)= f[k_1 \sum_{\substack{j=1 \\ j \ne i}}^{n} x_j(t-1)W_{ji} + k_2 x_i(t-1)] \tag{4}$$

Where $x_i(t)$ is the value of concept C_i at time t, $x_i(t-1)$ is the value of concept C_i at time $t-1$, $x_j(t-1)$ is the value of concept C_j at time $t-1$, and W_{ji} is the weight of the interconnection from C_j to C_i, and f is a threshold function. The parameter k_2 represents the proportion of the contribution of the previous value of the concept in the computation of the new value and the k_1 expresses the influence from the interconnected concepts in the configuration of the new value of the concept x_i. The two parameters k_1 and k_2 satisfy the equation:

$$0 < k_1, k_2 \le 1 \tag{5}$$

A more general and compact mathematical model for Fuzzy Cognitive Maps is proposed by the following equation:

$$\mathbf{X}(t)= f[k_1(\mathbf{W}^{\mathsf{T}}\, \mathbf{X}(t-1))+k_2\, \mathbf{X}(t-1))] \tag{6}$$

Therefore, equation 6 computes the new state vector $\mathbf{X}$, which results from the multiplication of the previous, at time t, state vector $\mathbf{X}$ by the edge matrix $\mathbf{W}$ and the adding of a fraction of the past values of concepts. The new state vector holds the new values of the concepts after the interaction among concepts of the map. The interaction was caused by the change in the value of one or more concepts.

It is proposed the values of two parameters to vary during the training period of the FCM, starting with a high value for parameter k_2, near to 1, and a low value for parameter, k_1 near to zero, and then to converge to equal values. Generally, the values of two parameters are dependent on each specific FCM.

2.3 FCM of Type III

Another formulation of FCMs will be presented and it is beyond the initial definition of Fuzzy Cognitive Maps, which do not allow any concept to influence itself (Kosko, 1986). Now, a concept can take into account its own past value with a weight w_{ii}. In this way, FCM will be close to Type II and the calculation rule will be similar to the equation 4:

$$x_i(t) = f[\sum_{\substack{j=1 \\ j\neq i}}^{n} x_j(t-1)W_{ji} + W_{ii}x_i(t-1)] \tag{7}$$

where x_i is the value of concept C_i at time t, $x_i(t-1)$ is the value of concept C_i at time $t-1$, $x_j(t-1)$ is the value of concept C_j at time $t-1$, and W_{ji} is the weight of the interconnection from C_j to C_i, W_{ii} is the weight with which the previous value of concepts participate in the calculation of the new and f is a threshold function.

A more compact form will be:

$$\mathbf{X}(t) = f(\mathbf{W}^{\mathsf{T}}\,\mathbf{X}(t-1)) \tag{8}$$

where $\mathbf{W}$ has nonzero diagonal elements. In equation 2, all diagonal elements of matrix $\mathbf{W}$ were zero.

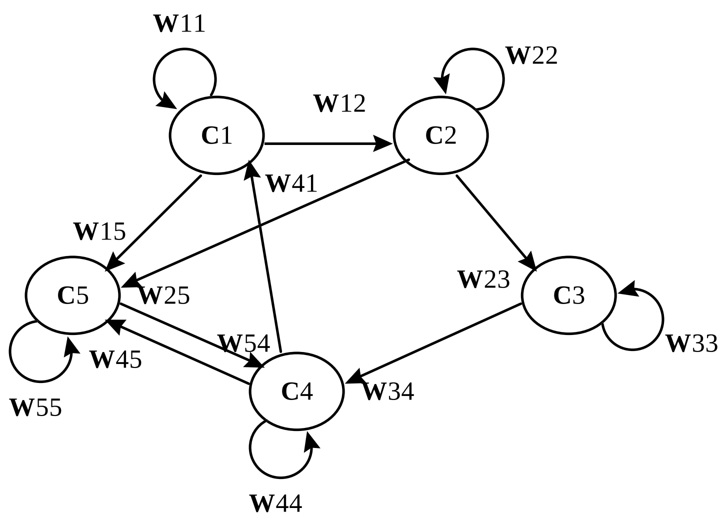

Figure 2. A Fuzzy Cognitive Map of Type III

This Type III of FCM must be examined in terms of his stability. There is a high possibility to be driven to a limit cycle or to be unstable, as there will be a constant increase in the value of each concept as a result of the influence that each concept has to itself. Fuzzy Cognitive Maps of type III will be useful to describe the behavior of some special systems under some circumstances.

3. New mathematical formulation of Fuzzy Cognitive Maps considering inputs to concepts.

3.1. General Description

A new type of Fuzzy Cognitive Maps is proposed, assuming that each concept has an external input, which influence the concept with a weight and it is taking into account in the calculation rule (figure 3). This type of FCMs are very similar to the Recurrent Neural Networks (Albertini and Pra, 1995), which are used as models whose parameters must fit to input/output data, minimizing a cost function and for which the controllability, observability and forward accessibility have been examined (Sontag and Sussmann, 1997; Albertini and Sontag, 1994).

Fuzzy Cognitive Map of type I with the assumption of the existence of external input to every node is considered. The value of each node of the map is calculated according to the following equation:

$$x(t+1) = f(Ax(t) + Bu(t)) \tag{9}$$

For the FCM of the figure 3, it is:

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ x_4(t+1) \\ x_5(t+1) \end{bmatrix} = f(\begin{bmatrix} 0 & 0 & 0 & w_{41} & w_{51} \\ w_{12} & 0 & w_{32} & 0 & 0 \\ 0 & w_{23} & 0 & 0 & 0 \\ 0 & 0 & w_{34} & 0 & 0 \\ 0 & 0 & 0 & w_{45} & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} + \begin{bmatrix} w_1 & 0 & 0 & 0 & 0 \\ 0 & w_2 & 0 & 0 & 0 \\ 0 & 0 & w_3 & 0 & 0 \\ 0 & 0 & 0 & w_4 & 0 \\ 0 & 0 & 0 & 0 & w_5 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \\ u_5(t) \end{bmatrix})$$

where $x_1(t)$ is the state of the node C_1 at time t and $x_1(t+1)$ is the state of the node C_1 at time t+1, this new state is calculated by the input $u_1(t)$ with the respective weight w_1 and the influence of the other nodes $w_{41}x_4(t) + w_{51}x_5(t)$.

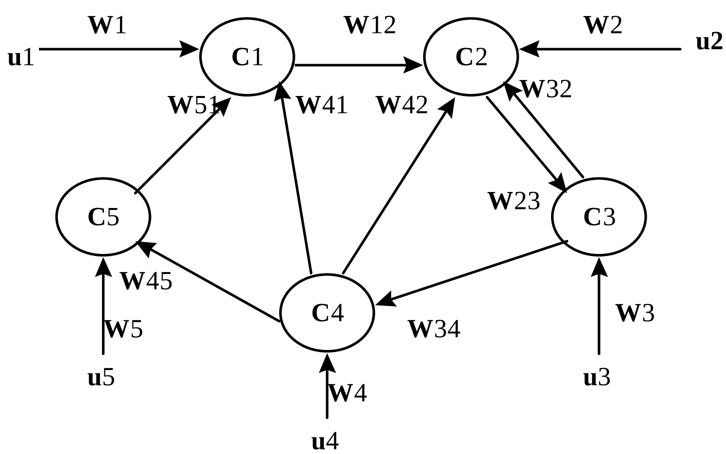

Figure 3. A simple Fuzzy Cognitive Map

This is the discrete time model of the system, with the assumption that values of concepts change step by step:

$$x(t+1) = f(Ax(t) + Bu(t)) \tag{10}$$

$$y(t) = Cx(t) \tag{11}$$

Generally, under nonlinearity conditions on f(.) and nondegenerity conditions on the matrix B, necessary and sufficient conditions for indentifiability, controllability and observability are given in terms of algebraic assumptions on the matrices A, B, C.

3.2 Controllability and Forward Accessibility for Discrete-Time FCM models

Fuzzy Cognitive Map system is said to be controllable if for every two states of the concept C_i the x_i^1 and x_i^2 there is a sequence of controls or inputs u[0],u[1],….u[k] in the discrete time case, or a control function u(t) for $t \in [0,T]$ in the continuous time case, which steers x_i^1 to x_i^2.

This notion is quite strong and it usually very hard to study for nonlinear systems as the examined FCM model which is characterized by hard nonlinearities. For this reason a proposed approach is the study for the weaker notion of forward accessibility.

A control system is said to be **forward accessible**, if for every initial state x_i^0, the set of points to which x_i^0 can be steered, contains an open subset of the state space.

The controllability for linear systems ($f(x)=x$ or the system $x(t+1)=Ax(t)+Bu(t)$) is satisfied if the $rank[\mathbf{A}-\lambda\mathbf{I},\mathbf{B}]=n$ for every $\lambda \in C$.

Albertini and Pra (1995) proved that under some conditions for $f(.)$ and $\mathbf{B}$, the system is forward accessible if $rank[\mathbf{A},\mathbf{B}]=n$.

The examined FCM model has dynamics that are described by the difference equation:

$$\Sigma: \quad x(t+1)=f(Ax(t)+Bu(t)) \qquad (13)$$
$$u(t)\in R^m,\ \ A \in R^{nxn}\ \ B \in R^{nxm}$$

In (Albertini and Pra,1995), the necessary conditions for the forward accessibility attribute of such systems have been described

3.2.1 Sufficient Condition I

Theorem

Let Σ a system of (13) where the function $f(.)$ and the matrix $\mathbf{B}$, satisfy the n-**IP** property (n-Independence property). Then Σ is forward accessible if and only if $rank[\mathbf{A},\mathbf{B}]=n$.

Definition

The n-**IP** property for function $f(.)$ and the matrix $\mathbf{B}$, is satisfied if :

1. f is differentiable and $f'(x)\neq 0$ for all $x \in R$;
2. for $b_i \neq 0$ for all $i=1,....,n$ where b_i $i=1,....,n$, the rows of the matrix $\mathbf{B}$;
3. For $1 \le k \le n$ let O_k be the set of all the subsets of $\{1,....n\}$of cardinality k and $a_1,....a_n$ arbitrary real numbers. Then the functions $\{g_I : I \in O_k\}$, $g_I : R^m \to R$ given by $g_I(u)=\prod_{i\in I} f'(a_i+b_i u)$ are linearly independent.

Special Case

The n-**IP** property is given as a joint property of function $f(.)$ and matrix $\mathbf{B}$, but in the framework of the FCM, function $f(.)$ is a given activation function, the sigmoid function. For this particular case, only the "genericity" condition is necessary for the **IP** property:

1. for $b_i \neq 0$ for all $i = 1,....,n$ and $|b_i| \neq |b_j|$ for every $i, j = 1,....,n$, $i \neq j$ for the rows of the matrix $\mathbf{B}$

3.2.2 Sufficient Condition II

Theorem
Let Σ a system of (13) where the function $f(.)$ and the matrix $\mathbf{B}$, satisfy the n-**WIP** property (n-Weak Independence property). If there exists a matrix $\mathbf{H} \in R^{mxn}$ such that:

a. The matrix $(\mathbf{A} + \mathbf{BH})$ is invertible
b. the rows of the matrix $[(\mathbf{A} + \mathbf{BH})^{-1}\mathbf{B}]$ are all nonzero, then Σ is forward accessible.

Definition
The n-**WIP** property for function $f(.)$ and the matrix $\mathbf{B}$, is satisfied if :

1. f is differentiable and $f'(x) \neq 0$ for all $x \in R$;
2. for $b_i \neq 0$ for all $i = 1,....,n$ where b_i $i = 1,....,n$, the rows of the matrix $\mathbf{B}$;
3. Let $a_1,....a_n$ be arbitrary real numbers, then the functions from R^m to $R(f'(a_i + b_i u))^{-1}$ for $i = 1,...n$ are linearly independent.

3.3 Future research

In the case that not all the concepts of Fuzzy Cognitive Maps have nonzero inputs (every row of $\mathbf{B}$ not to be nonzero), provided that $\mathbf{A}$ is appropriately chosen it must be mentioned that the system may be controllable. This is very important and it is subject of future research.

Moreover, this work can accompany with research results presented in (Kosko, 1997) where stability of Fuzzy Cognitive Maps has been examined in terms of the eigenvalues of the weight connection matrix $\mathbf{A}$. It must be mentioned that in (Stylios and Groumpos, 1998) proofs of theorems are presented in detail.

4. Practical Process Control Problem

This control problem has presented in (Stylios *et al.*, 1997b) and it describes the application of Fuzzy Cognitive Map model in a well-known problem in process industry is shown. Through this example it will become clear how a Fuzzy Cognitive Map is constructed, how concepts are chosen, how are assigned values to the interconnections between concepts and eventually how this FCM models and controls a process.

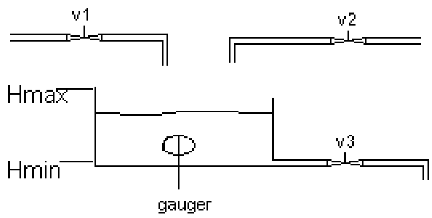

Figure 4. The system of a simple process

The considered system consists of one tank and three valves that influence the amount of liquid in the tank; figure 4 shows an illustration of the system. Valve 1 and valve 2 empty two different kinds of liquid into tank 1, during the mixing of the two liquids a chemical reaction takes place into the tank. Into the tank there is an instrument tool that measures the specific gravity of the liquid that is produced into tank and when gravity takes value in the range between (G_{max}) and (G_{min}), this means that the desired liquid has been produced into tank. Moreover there is a limit on the height of liquid into tank, which cannot excess an upper limit (H_{max}) and a low limit (H_{min}). So the control target is to keep these variables in the middle of their range of values:

$$\begin{aligned} G_{min} \le G \le G_{max} \\ H_{min} \le H \le H_{max} \end{aligned} \tag{16}$$

In order to construct a Fuzzy Cognitive Map, which will model and control this simple system, the concepts of the map must be determined. Concepts will stand for the variables and states of the plant, as it is the height of liquid in the tank or the state of the valve. So a primitive FCM will have five concepts and later any new concept, which will improve model and control of the system, can be added:

Concept1 The amount of the liquid which tank1 contains. This amount is dependent on valve 1, valve 2 and valve 3.
Concept2 The state of the valve 1 (closed, open, partially opened).
Concept3 The state of the valve 2 (closed, open, partially opened).
Concept4 The state of the valve 3 (closed, open, partially opened).
Concept5 The reading on the instrument of specific gravity.

The real value of the physical magnitude that each concept represents is transformed in the range [0,1] where concepts take values. After having selected the concepts that can represent the model of the system and its operation behavior, the interconnections between concepts must be decided. At first, it is decided for each concept with which other concept it will be connected. Then, the sign and weight of each connection is determined. All this procedure has been done by a group of experts who have experience on the system's operation.

The connections between concepts are:

Event1 It connects concept 2 (valve 1) with concept1 (amount of liquid in the tank). It relates the state of the valve 1 with the amount of the liquid in tank.
Event2 It relates concept 3 (valve 2) with concept1; valve 2 causes the increase or not of the amount of liquid in tank.
Event3 It connects concept 4 (valve 3) with concept 1; the state of valve 3 causes the decrease or not of the amount of liquid into tank.
Event4 It relates concept1 with concept 2; when the height of the liquid in tank is high, valve 1 (concept 2) needs closing and so the amount of incoming liquid into tank is reducing.
Event5 It connects concept 1 (tank) with concept 3; when the height of the liquid in tank is high, the closing of valve 2 (concept 3) reduces the amount of incoming liquid.

Event6 It connects concept 5 (the specific gravity) with concept 4 (valve 3). When the quality of the liquid in the tank is the appropriate, valve 3 is opened and the produced liquid continues to another process.

Event7 It shows the effect of concept 1 (tank) into concept 5 (specific gravity). When the amount of liquid into tank is varied, this influence in the specific gravity of the liquid.

Event8 It relates concept 5 (specific gravity) with concept 2 (valve 1), when the specific gravity is very low then valve 1 (concept 2) is opened and liquid comes into tank

Figure 5 shows the FCM that is used to describe and control this simple system, the initial value of each concept, the interconnections and the weights between concepts are illustrated. The values of concepts correspond with the real measurement of the physical magnitude. The values of the each event (it represent the weight of each interconnection between concepts) has been determined by the experts who designed the map. Experts have observed the influence of each concept to the others in the real experimental system, and the assigned linguistic weight values for each interconnection, which were transformed in fuzzy values.

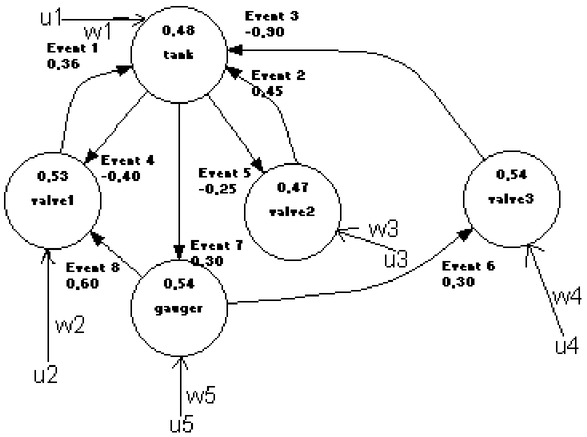

Figure 5. The Fuzzy Cognitive Map who controls the process.

The Fuzzy Cognitive Map can be described by the following equation:

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ x_4(t+1) \\ x_5(t+1) \end{bmatrix} = f(\begin{bmatrix} 0 & w_{21} & w_{31} & w_{41} & 0 \\ w_{12} & 0 & 0 & 0 & w_{52} \\ w_{13} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & w_{54} \\ w_{15} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} + \begin{bmatrix} w_1 & 0 & 0 & 0 & 0 \\ 0 & w_2 & 0 & 0 & 0 \\ 0 & 0 & w_3 & 0 & 0 \\ 0 & 0 & 0 & w_4 & 0 \\ 0 & 0 & 0 & 0 & w_5 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \\ u_5(t) \end{bmatrix})$$

It is assumed that all the concepts have inputs from the real system. Input for concept 1 is the desired height of liquid in the tank and input for concept 5 is the desired specific gravity of the produced liquid in the tank. So the equation will be:

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ x_4(t+1) \\ x_5(t+1) \end{bmatrix} = f(\begin{bmatrix} 0 & 0.36 & 0.45 & -0.90 & 0 \\ -0.40 & 0 & 0 & 0 & 0.60 \\ -0.25 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.30 \\ 0.30 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \\ u_5(t) \end{bmatrix})$$

Outputs of this Fuzzy Cognitive Map are the values of concepts that represent the state of valves, so:

$$\begin{bmatrix} y_1(t+1) \\ y_2(t+1) \\ y_3(t+1) \\ y_4(t+1) \\ y_5(t+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ x_4(t+1) \\ x_5(t+1) \end{bmatrix}$$

The squeezing function, which is used, is $f(x) = \frac{1}{1+e^{-\lambda x}}$ and the matrix $\mathbf{B}$ with the weights w_{ii}, will satisfy the **IP** property.

It can be assumed that the inputs to the Fuzzy Cognitive Map are resulting from the transformation of the measurements of the variables of the real system and/or their desired values.

In the case that there are less than n inputs then some of the rows of $\mathbf{B}$ will be zero and the controllability of FCM must be examined according to the relation of matrices $\mathbf{A}$ and $\mathbf{B}$.

5. Conclusions

In this paper different mathematical formulations of Fuzzy Cognitive Maps have been presented and new types of Fuzzy Cognitive Maps have been examined. The new proposed type of FCM has a mathematical representation very close to Recurrent Neural Networks, exploiting this attribute of Fuzzy Cognitive Maps, the examination of Forward Accessibility and Controllability for Fuzzy Cognitive Maps is presented. Some initial research results are discussed that they may open new directions in the use of Fuzzy Cognitive Maps in Control Systems.

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