Introducing Interval Analysis in Fuzzy Cognitive Map Framework

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Abstract. Fuzzy Cognitive Maps (FCMs) is a graphical model for causal knowledge representation. FCMs consist of nodes-concepts and weighted edges that connect the concepts and represent the cause and effect relationships among them. FCMs are used in complex problems involving causal relationships, which often include feedback, and where qualitative rather than quantitative measures of influences are available. They have used for decision support to determine a final state given a qualitative initial knowledge for nodes and weighted edges. A first study on introducing Interval analysis in the FCM framework has been attempted and it is presented in this work. Here a new structure for FCM is proposed with interval weights and a new method for processing interval data input for FCMs is proposed.

Keywords: Fuzzy cognitive maps, Interval analysis.

1 Introduction

FCMs are qualitative models for a system, consisted of variables and the causal relationships between those variables [1]. They are useful for knowledge representation and processing for highly complicated domains [2], [3]. The usefulness of FCMs is highlighted for dynamic feedback systems for which conventional rule-based expert systems are inadequate [4]. Experts design and determine the nodes and weights of FCM, for each problem domain [5], [6]. Fuzzy variables are used for concepts and weights. Here it is introduced the treatment for nodes and the weights as interval numbers that lie within the range provided by the experts, rather than as fuzzy sets.

Interval analysis is a deterministic way of representing uncertainty by replacing a number with a range of values [7], [8]. It introduced to deal with numerical errors which occurred in mathematical computations performed on digital computers. Interval analysis has been used in many branches of mathematics, including numerical analysis, probability and logic [9], [10] and [11].

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In [12], Muata and Bryson used interval pairwise comparison techniques for generating consistent subjective estimates for the magnitudes of causal relationships in FCMs. Here it is introduced interval computations in all the FCM architecture.

2 A Brief Description of Fuzzy Cognitive Maps

A FCM is composed of nodes that represent the factors most relevant to the decision environment and weighted arrows that indicate the causal relationships among factors. One factor (variable) can have a positive or negative effect on another. Arrows have different numerical strengths. Experts describe their understanding of the relationships among the key factors. The directional influences are presented as all-or-none relationships, so the FCMs provide qualitative as well as quantitative information about these relationships [1].

Generally, the value of each node is calculated, computing the influence of the interconnected nodes to the specific node, by applying the following calculation rule:

$$A_i^{(k+1)} = f(A_i^{(k)} + \sum_{\substack{j \neq i \\ j=1}}^{N} w_{ji} \cdot A_j^{(k)}) \tag{1}$$

where $A_i^{(k+1)}$ is the value of node C_i at time $k+1$, $A_j^{(k)}$ is the value of node C_j at time k, w_{ji} is the weight of the interconnection from node C_j towards node C_i and f is the sigmoid threshold function. When a node(s) is stimulated, then the resulting activities can resonate through other nodes on the FCM until equilibrium is reached.

In the proposed methodology, both nodes and edges are fuzzy sets and are bounded in ranges provided by the inference method and the related membership functions. At follows, three different types for assessing FCM concepts and weights as interval values are introduced and a new mathematical form for FCM framework is proposed.

3 Introducing Interval Analysis in FCM Framework

Interval Fuzzy Cognitive Map (IFCM) is a FCM where its concepts (inputs, outputs) and weight take interval values. Interval FCMs are formed by processing units called Interval Concepts. An interval FCM is formed by three functions:

– Normalizer Function (T): This function analyzes the input nature and normalizes it in order to take only interval inputs.
– Sum Function (Σ): This function is the same as the sum function in the calculation for traditional FCMs.
– Activation Function: This function could be any interval derivable linear function, which restricts the output to interval values between [-1,1] or [0,1].

Interval FCMs can be classified in three different types depending on the concept's nature and nature of weights (fuzzy-interval). In the following subsection we describe the three different approaches for assessing FCMs using Interval Computations.

3.1 Type I of Interval FCM

In this approach, the initial concept values are interval numbers; the weights take crisp values and the output concepts take interval numbers. Fig. 1 represents the IFCM type I, with crisp weight values and interval values for input-output concepts.

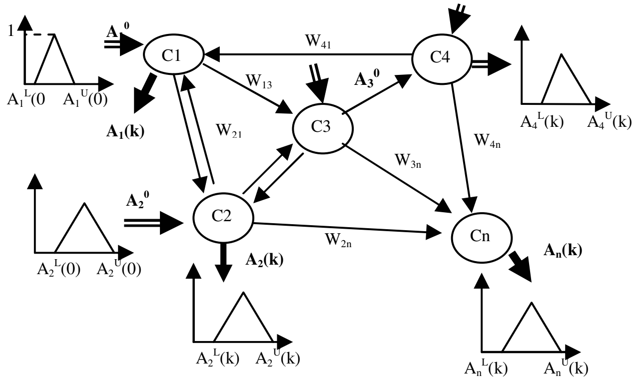

Fig. 1. Interval FCM model I with interval input-output concepts and crisp weigh++ts

The output concept value A_p, for a concept C_p is calculated by equations (2), (3):

$$A_{pi}^{L}(k) = A_{pi}^{L}(k-1) + \sum_{\substack{i=1,\\ wij \geq 0}}^{n} w_{ji} \cdot A_{j}^{L}(k-1) + \sum_{\substack{i=1,\\ wij \prec 0}}^{n} w_{ji} \cdot A_{j}^{U}(k-1) \tag{2}$$

$$A_{pi}^{U}(k) = A_{pi}^{U}(k-1) + \sum_{\substack{i=1,\\ wij \geq 0}}^{n} w_{ji} \cdot A_{j}^{U}(k-1) + \sum_{\substack{i=1,\\ wij \prec 0}}^{n} w_{ji} \cdot A_{j}^{L}(k-1) \tag{3}$$

Where the interval output concept is determined from the form:

$$A_p(k) = \left[A_p^L, A_p^U\right] = \left[f(A_{pi}^L), f(A_{pi}^U)\right] \tag{4}$$

3.2 Type II for Interval FCM

Type II of IFCM uses interval values for weights and crisp values for input concepts. The derived output concept values are calculated using equation (5):

$$A_{pi}(k) = \left[A_{pi}^L(k), A_{pi}^U(k)\right] = A_{pi}(k-1) + \sum_{i=1}^{n}\left[w_{ij}^L, w_{ij}^U\right]\cdot\left[A_j^L(k-1), A_j^U(k-1)\right] \tag{5}$$

The initial concept values $\mathbf{A}^0$ are crisp values derived from measurements and represent real numerical data.

3.3 Type III for Interval FCM

In this approach, all the FCM concepts and weights are determined as interval numbers producing an Interval FCM framework. The output concepts values are calculated through the equation (6), considering multiplication and addition of intervals.

$$A_{pi}(k) = \left[A_{pi}^{L}(k), A_{pi}^{U}(k) \right] = \left[A_{pi}^{L}(k-1), A_{pi}^{U}(k-1) \right] + \sum_{i=1}^{n} \left[w_{ji}^{L}, w_{ji}^{U} \right] \cdot \left[A_{j}^{L}(k-1), A_{j}^{U}(k-1) \right] \quad (6)$$

4 Conclusions and Future Directions

In this paper, interval analysis has been introduced to assess FCM's concepts and weights. This work proposes a new framework of Fuzzy Cognitive Map that updates the traditional Fuzzy Cognitive Map and has better characteristics. Only, Muata and Bryson [11] referred to interval pairwise comparison techniques for generating consistent magnitudes of FCM causal relationships. This paper proposes the inclusion of interval mathematics in the structure of the FCM, it is expected that the performance of the FCM with interval mathematics could be better to deal with interval types of input data eliminating numerical errors.

Our future work is directed towards the implementation of Interval Fuzzy Cognitive Maps in real problem from different scientific domains, proposing training algorithms for IFCM and comparing the results with other approaches. Furthermore, future research will be directed to fuzzy training algorithms for IFCMs in order to compare them with the recently proposed learning techniques for FCMs [5], [6].

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