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FUZZY COGNITIVE MAPS: A SOFT COMPUTING TECHNIQUE FOR INTELLIGENT CONTROL

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Abstract. This paper describes a soft computing technique for modelling and controlling systems, Fuzzy Cognitive Maps. The description, representation and models of FCM are examined thoroughly, a FCM model is proposed and their characteristics and advantages are presented, and a development algorithm is described. Fuzzy Cognitive Maps and similar soft computing techniques may contribute to develop more sophisticated systems.

Key words. Soft Computing, Fuzzy Cognitive Maps, Modeling, Complex Systems

1. INTRODUCTION

Soft computing is an emerging field that combines and synergies advanced theories and technologies such as Fuzzy Logic, Neural Networks, Probabilistic Reasoning and Genetic Algorithms. Soft computing provides a hybrid flexible computing technology that can solve real world problems. Soft computing includes not only the previously mentioned approaches but also useful combinations of its components. Soft computing includes Neurofuzzy systems, Fuzzy Neural systems, and usage of Genetic Algorithms in Neural Networks and Fuzzy Systems and many other hybrid methodologies. The main characteristic of soft computing is its "adaptive behaviour" so that systems adapt to users' perceptions. Soft Computing techniques have been successfully applied in image recognition area, signal processing, and market forecasting. Fuzzy logic and fuzzy sets play a core role in soft computing by providing the representation of uncertainty, granulation and bias of perception [5][9].

The evolution of control theory and the development of new methodologies and techniques have created a trend and a need for more advanced systems. When controlling a system has achieved, the need for more sophisticated systems with more autonomy arises, and novel methodologies are emerged in order to lend to Intelligent Control. Soft Computing theories combines and synergies advanced theories and novel technologies in order to bring intelligence into systems.

Real world problems are typically difficult to model, ill-defined systems and with a large-scale solution space. For such kind of systems, precise models are impractical, too expensive or non-existent. On the other hand most of the available information is in the form of empirical prior knowledge and input-output data. Nature of knowledge and problems bring up the need for approximate reasoning methods that will handle imperfect information [12].

Fuzzy Cognitive Maps (FCM) belong to the soft computing approaches and they originate from the implementation of fuzzy logic techniques to represent knowledge and behaviour of a system using a network of interconnected nodes. Nodes of Fuzzy Cognitive Map represent concepts of the behaviour and model of the system. Fuzzy Cognitive Map can be used to model and represent the behaviour of a simple and complex system. FCMs aim to mimic human reasoning, they capture and emulate the nature of human being in describing, presenting and modelling systems, including tolerance for imprecision and granulation of information. They are adaptive and intelligent systems in nature and they do not belong to the conventional hard computing methods [7][8].

2. FUZZY COGNITIVE MAPS

Fuzzy Cognitive Maps (FCM) is a modelling technique, which is originated from the combination and synergism of Fuzzy Logic and Neural Networks.

2.1 Graphical Representation of FCM

Fuzzy Cognitive Maps are illustrated graphically as a signed fuzzy graph with feedback, consisting of nodes and weighted interconnections. Nodes of the FCM stand for concepts that are used to describe main behavioural characteristics of the system. Signed and weighted arcs represent the causal relationships that exist among concepts. This graphical representation illustrates which concept influences other concepts, showing the interconnections between concepts. In conclusion, an FCM is a fuzzy-graph structure, which allows systematic causal propagation, in particular forward and backward chaining [12].

There are no limitations or directions on the connections among concepts, a concept can be connected with all the other concepts; so cycles of cause and effect, direct and indirect feedback can be included in the FCM structure. Accepting this kind of interconnections among concepts, systems of any dynamic can be modelled with a FCM.

Concepts reflect attributes, characteristics, qualities, and senses of the system. Interconnections among concepts of FCM signify the cause and effect relationship that a concept has on the others. These weighted interconnections represent the direction and degree with which concepts influence the value of the interconnected concepts. Fig. 1 illustrates the graphical representation of Fuzzy Cognitive Maps. FCM is consisted of N concepts that are connected. Each connection is characterized by a correlation weight that describes influence of one concept on the other.

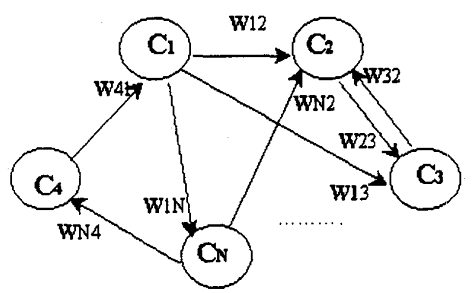

Fig.1.Graphical illustration of Fuzzy Cognitive Maps

2.2 Models of FCM

FCM is consisted of nodes that are called concepts and they represent a characteristic, factor, variable, state, output or event of the system that FCM models its operation and behaviour. Interconnections among concepts express the cause and effect relationship that exists between two concepts, this influence can be direct or indirect. Interconnections describe the influence that has the variation on the value of one concept in the configuration of the value of the interconnected concept. This causal relationship is characterized with vagueness, because of its nature, as it represent the influence of one qualitative factor on another one and because of its determination, using linguistic variables.

Definition 2.1 *Direction of correlation between two concepts.*

The causal relationship between two concepts of FCM can have the following directions:

Concept C_i influence concept C_j and there is a connection between $i \longrightarrow j$, so $\delta_{i,j}=1$. Either there is a connection with the reverse direction $j \longrightarrow i$ when concept C_j influences concept C_i and so $\delta_{j,i}=1$, or there is no connection between these two concepts.

$$i,j=\begin{Bmatrix} \delta_{i,j}=1 \\ \delta_{j,i}=1 \\ 0 \end{Bmatrix} \qquad □$$

Definition 2.2 *Sign of correlation between two concepts.*

Correlation between two concepts can be positive or negative, or no correlation:

i) $W_{ij}>0$, it means that when value of concept C_i increases, the value of concept C_j increases, and when value of concept C_i decreases then value of concept C_j decreases.

ii) $W_{ij}<0$, it means that when value of concept C_i increases, the value of concept C_j decreases, and when value of concept C_i decreases then value of concept C_j increases.

iii) $W_{ij}=0$, there is no relationship of concept C_i to concept C_j. □

Definition 2.3. *Degree of correlation between two concepts*

The value of the weight for the interconnection W_{ij} between concept C_i and concept C_j expresses the degree of correlation of the value of one concept on the calculation of the value of the interconnected concept. Numerical values of weights W_{ij} belong to the interval [-1,1]. □

Comment

The correlation between two concepts declares the cause and effect relationship between two concepts. Moreover, a concept can not causes itself and there is no causal relationship between a concept and itself, so for all the concepts of FCM there will be $W_{ii}=0$

Experts who have observed the system operation perform designing and development of Fuzzy Cognitive Map, they have experience on its model and they have created a mental model of the system and its behaviour. Within FCM development procedure, experts determine the direction of correlation between two concepts (definition 2.1), the sign (definition 2.2) and the degree of correlation (definition 2.3)

After the development of Fuzzy Cognitive Map the operation of the system can be simulated. At every simulation step, the values of concepts are calculated with the following mathematical procedure. This calculation rule calculates the value of each concept taking under consideration the influence of the interconnected concepts with the corresponding weights [11]:

$$A_i^t = f\left(\sum_{\substack{j=1 \\ j \neq i}}^{N} A_j^{t-1} W_{ji}\right) \tag{1}$$

The value of concept C_i at time t is A_i^t. This value is depended on the values of the interconnected concepts at time $t-1$ multiplying with the corresponding weight W_{ji}. The aggregated influence is calculated and this sum passes through a threshold function f that gives as output a value for concept C_i in the interval [0,1], where the N concepts of Fuzzy Cognitive Map take values

Threshold function

The threshold function f that is used in the calculation of FCM is the sigmoid function. This threshold function transforms its input in the interval [0,1]:

$$f(x) = \frac{1}{1+e^{-\lambda x}} \tag{2}$$

Where the parameter $\lambda \succ 0$ determines the curve of the continuous sigmoid function f.

2.3 New Mathematical Model for FCM

A new mathematical model has been proposed to calculate the value of each concept at time t. It is proposed the use of the previous value of each concept in the sum of equation (1).

$$A_i^t = f\left(\sum_{\substack{j=1 \\ j \neq i}}^{N} A_j^{t-1} W_{ji} + A_i^{t-1}\right) \tag{3}$$

Namely, A_i^{t-1} is the value of concept C_i at time $t-1$.

The use of the past value of each concept in the calculation of its sequential value, influences the Fuzzy Cognitive Map converge. FCM values of concepts reach the equilibrium region slighter with this new method. Moreover each concept has memory of one step and so the transition from one value to another is smoother.

2.3.1 Generalized Method

A generalized method is proposed for the calculation of new values for each concept. A coefficient is introduced to determine the contribution of the past value of each concept in the calculation of the new value for each concept. Using this coefficient and the past value of each concept, problems of stability and controllability can be examined with more suppleness, as the bias of the weight matrix is not zero but can be determined by the expert. Introducing the coefficient γ in equation (3) it is:

$$A_i^t = f\left(\sum_{\substack{j=1 \\ j \neq i}}^{N} A_j^{t-1} W_{ji} + \gamma A_i^{t-1}\right) \tag{4}$$

Where, the coefficient γ represents the participation of the past value of each concept in the calculation of the new value of concept. Coefficient γ can take values in the interval $0 \leq \gamma \leq 1$, and can vary according to the desirable contribution of the previous value in different circumstances, so value of γ can change within time so $\gamma = \gamma(t)$. It is proposed during the training period of FCM, the value of γ to be near the one, because the greater the influence of past value the smoother the change of the new value; and later during the permanent operation of the Fuzzy Cognitive Map will be posed near to 0.1.

2.4 Overall Calculation Rule

Assuming that Fuzzy Cognitive Map is consisted of N concepts, then there will be a matrix **A** with dimension $1xn$, that gathers the values of N concepts and there will be a weight matrix **W** of nxn, where every element e_{ij} of matrix **W** represents the weight W_{ij} of the connection from C_i towards C_j.

Thus equation (3) in a more concrete formula, which will describe the calculation of values of all the concepts of Fuzzy Cognitive Map, will be :

$$\mathbf{A}^t = f(\mathbf{A}^{t-1}\mathbf{W} + \mathbf{A}^{t-1}) \tag{5}$$

Respectively, equation (4) in a concrete formula will be:

$$\mathbf{A}^t = f(\mathbf{A}^{t-1}\mathbf{W} + \gamma\ \mathbf{A}^{t-1}) \tag{6}$$

3. SELF-FEED FUZZY COGNITIVE MAP

At first, it was assumed that Fuzzy Cognitive Map concepts couldn't influence themselves as there is no cause and effect relationship between a concept and itself and so the weight W_{ii} is zero. So at the weight matrix **W** the diagonal elements are all zero.

Equation 6 is used to calculate the value of concepts taking under consideration the past value of each concept. The weight matrix **W** gathers only the weights of the interconnections among concepts. So there will be:

$$\mathbf{A}^t = f[(\mathbf{A}^{t-1}\mathbf{W} + \gamma\mathbf{A}^{t-1})] \quad ή$$
$$\mathbf{A}^t = f[\mathbf{A}^{t-1}(\mathbf{W} + \gamma\mathbf{I})] \quad ή$$
$$\mathbf{A}^t = f(\mathbf{A}^{t-1}\mathbf{W}^{new}) \qquad (7)$$

Namely, the weight matrix, $\mathbf{W}^{new}$ has all the diagonal elements not zero but it is $W_{ii}=\gamma$ and all the rest elements represent the weights of the interconnections among different concepts. The weight matrix $\mathbf{W}^{new}$ is :

$$\mathbf{W} = \begin{bmatrix} W_{11} & W_{12} & 0 & 0 & W_{15} \\ 0 & W_{22} & W_{23} & 0 & W_{25} \\ 0 & 0 & W_{33} & W_{34} & 0 \\ W_{41} & 0 & 0 & W_{44} & W_{45} \\ 0 & 0 & 0 & W_{54} & W_{55} \end{bmatrix}$$

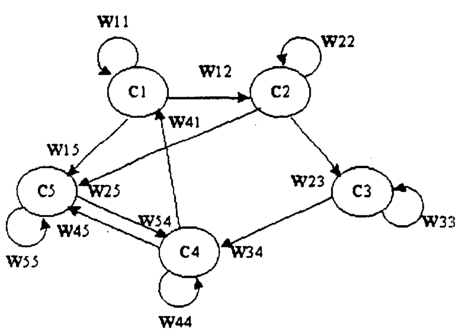

Fig.2. Self-feed Fuzzy Cognitive Map

3.1 Self-feed FCM and Other Models.

Self-feed Fuzzy Cognitive Maps have many advantages. Using the past value of each concept in the calculation of the new value, our system has a memory of one step. We will illustrate our results using an example that was presented at [4]. The weight matrix for this FCM is:

$$\mathbf{W} = \begin{bmatrix} 0 & 0 & 0.6 & 0.9 & 0 & 0 & 0 \\ 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.9 \\ 0 & 0 & 0 & 0 & 0 & -0.9 & 0.9 \\ -0.3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3 & 0 \end{bmatrix}$$

The following initial values of concepts will be the initial values of concepts and equation 2 will be used to calculate the values of concepts and the results are presented at Table 1 and illustrated at fig.3.

$\mathbf{A_0}$= [0.90 0.01 0.30 0.73 0.80 0.20 0.50]

Table 1. The values of concepts of FCM using the classical method for 10 simulation steps

	C1	C2	C3	C4	C5	C6	C7
1	0.90	0.010	0.3	0.73	0.8	0.2	0.5
2	0.485	0.552	0.631	0.692	0.567	0.361	0.798
3	0.486	0.608	0.572	0.607	0.638	0.432	0.756
4	0.482	0.598	0.572	0.607	0.626	0.413	0.754
5	0.483	0.598	0.571	0.606	0.626	0.416	0.752
6	0.483	0.598	0.572	0.607	0.625	0.416	0.752
7	0.483	0.598	0.572	0.607	0.625	0.416	0.752
8	0.483	0.598	0.572	0.607	0.625	0.416	0.752
9	0.483	0.598	0.572	0.607	0.625	0.416	0.752
10	0.483	0.598	0.572	0.607	0.625	0.416	0.752

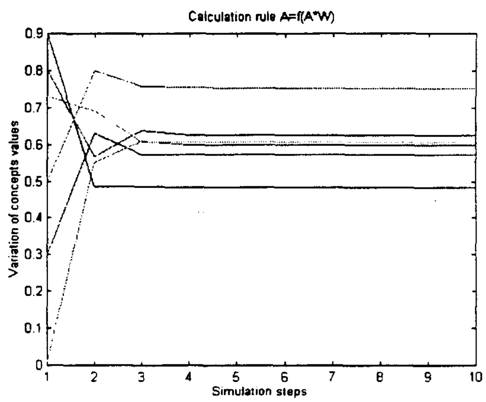

Fig.3. The variation of values of concepts for the classical calculation method.

For the self-feed model, equation (7) is used to calculate the values of concepts. For this example it is assumed that $W_{ii} = \gamma = 1$, and the new weight matrix will be:

$$\mathbf{W} = \begin{bmatrix} 1 & 0 & 0.6 & 0.9 & 0 & 0 & 0 \\ 0.1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 1 & 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0.9 \\ 0 & 0 & 0 & 0 & 1 & -0.9 & 0.9 \\ -0.3 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3 & 1 \end{bmatrix}$$

Comparing the values of concepts and their plots it can be inferred that at the classical method and self-feed method values of concepts reach their region of converge after 6 and 8 simulation steps respectively. Another advantage of self-feed method is that the system reaches the region of converge (eg. values for concept C_1 above 0.6) after the second step that means at 40% of the number of steps but with the other method value of concept C_1 just reaches this region.

Table 2. The values of concepts for self-feed FCM for 10 simulation steps

	C1	C2	C3	C4	C5	C6	C7
1	0.900	0.010	0.3	0.73	0.8	0.2	0.5
2	0.698	0.554	0.698	0.823	0.744	0.408	0.867
3	0.652	0.739	0.753	0.810	0.797	0.499	0.907
4	0.640	0.780	0.758	0.801	0.814	0.513	0.913
5	0.637	0.787	0.758	0.798	0.817	0.513	0.914
6	0.636	0.788	0.757	0.797	0.817	0.513	0.914
7	0.636	0.789	0.757	0.797	0.817	0.512	0.914
8	0.636	0.789	0.757	0.797	0.817	0.512	0.914
9	0.636	0.789	0.757	0.797	0.817	0.512	0.914
10	0.636	0.789	0.757	0.797	0.817	0.512	0.914

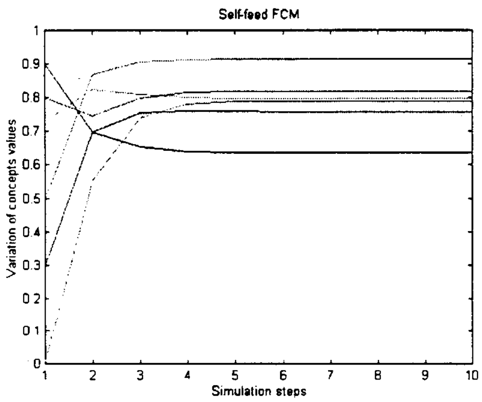

Fig.4.The variation of values of concepts for the self-feed FCM with $W_{ii} = \gamma = 1$.

Using Self-feed Fuzzy Cognitive Map the weights W_{ii} of the weight matrix can be selected. User can propose different self-feed weights for each concept depending on its own characteristics, making the model more flexible. The selection of self-feed weights can be done after the development of the system and after the training period of FCM. Selecting the appropriate self-feed weights the regions of converge can be determined.

4. FCM DEVELOPMENT

The methodologies for development and construction of Fuzzy Cognitive Map have great importance for FCMs utilization in the modelling of systems. FCM represents the human knowledge on the operation of the system and experts develop FCMs using their experience and knowledge on the system. Construction methodologies rely on the exploitation of experts' experience on system's model and behaviour. Experts determine the number and characteristics of concepts and the interrelationships among concepts using the following algorithm[11].

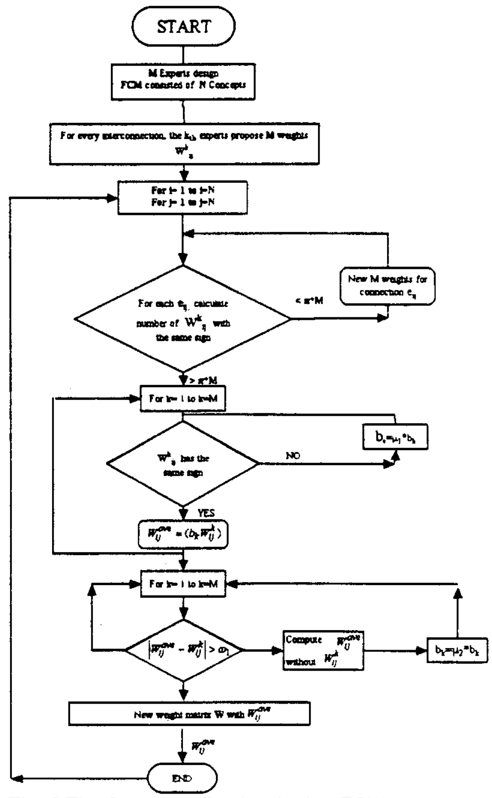

Fig. 5 The flow chart for developing FCM

It was supposed that there are M experts to develop the FCM. They know the main factors that determine the behaviour of the system; each one of these factors can be represented by a concept. Experts determine the number N and the kind of concepts according to their experience. Experts know, which elements of the system influence other elements; for the corresponding concepts they determine the negative or positive effect of one concept on the others, with a fuzzy degree of causation using the definitions 2.1, 2.2 and 2.3. For every expert the corresponding credibility weight b_k is posed equal to one. Every k_{th} expert of the M have proposed a weight for each interconnection, thus, for each interconnection there are M weights that are examined. At first, the sign of the M weights is examined, if the number of weights with the same sign is less than an acceptable percentage $\pi * M$, that means that experts have to assign new weights for this interconnection, and credibility of experts who proposed wrong signed weights is decreased $b_k = \mu_1 * b_k$. For weights with the same sign the average weight W_{ij}^{ave} is calculated. If the value of weight W_{ij} is far than an accepted distance ω_1 from the W_{ij}^{ave}, this weight is disregarded and the credibility of the corresponding experts is decreased by $b_k = \mu_2 * b_k$. All the weighted interconnections are examined with this procedure [14].

5. NEURAL NETWORK NATURE OF FUZZY COGNITIVE MAPS

The development and construction of FCMs is based on their fuzzy nature. Neural Network learning algorithms can be used to adjust the degree of causal relationship among concepts. Parameter learning of FCM concerns the updating of connection weights among concepts. The assigning of weights can be based on learning algorithms [13]. The development of FCMs is mainly based on experts who determine the concepts and weighted interconnections among concepts. This methodology may lead to a distorted model of the system because human factor is not always reliable. To refining the model of the system, learning rules are used to adjust weights of FCM interconnections.

The Differential Hebbian learning algorithm has been proposed [8] for FCMs. The Differential Hebbian learning law adjusts the weights of the interconnection between concepts. It grows a positive edge between two concepts if they both increase or both decrease and it grows a negative edge if values of concepts move in opposite directions. Adjusting differential Hebbian learning rule in the framework of Fuzzy Cognitive Map, the following rule is proposed to calculate the derivative of the weight between two concepts.

$$\dot{w}_{ji} = -w_{ji} + s(A_j^t)s(A_i^t) + \dot{s}(A_j^{t-1})\dot{s}(A_i^{t-1}) \quad (8)$$

Where $S(x) = \frac{1}{1+e^{-\lambda x}}$ and $\dot{w}_{ji}$ is the gradient of the weight interconnection from concept C_j towards C_i, which is depend on the past value A_i^{t-1} of concept C_i and the new value of A_j^{t-1} of concept C_j.

Learning rules for Fuzzy Cognitive Maps need more investigation. These rules will give FCMs useful characteristics such as the ability to learn arbitrary non-linear mappings, capability to generalize to situations, the adaptivity and the fault tolerance capability.

6. CONCLUSIONS

FCM have been implemented in discipline scientific areas but there is not enough theoretical research on this soft computing technique [1][2][10]. At this paper different formulations and new types of Fuzzy Cognitive Maps have been examined. New methodologies based on Fuzzy Logic approach for develop Fuzzy Cognitive Maps have been already investigated [3]. On the other hand there is a need for investigation of learning algorithms and techniques that make FCM independent of human experts who are involved in the developing FCM structure.

7. ACKNOWLEDGEMENT

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