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Automatizing the detection of rotor failures in induction motors operated via soft-starters

George Georgoulas, Petros Karvelis, Chrysostomos D. Stylios
Department of Computer Engineering, TEI of Epirus
Arta, Greece
pkarvelis@kic.teiep.gr, georgoul@kic.teiep.gr, stylios@teiep.gr

Jose Alfonso Antonino-Daviu, Jesús Corral-Hernández
Instituto de Ingeniería Energética
Universitat Politècnica de València
Valencia, Spain
joanda@die.upv.es, jecorher@doctor.upv.es

Ioannis P. Tsoumas
Larges Drives, Products R&D Department
Siemens Industry Sector - Drive Technologies
Nuremberg, Germany
ytsoumas@ieee.com

Vicente Climente-Alarcón
Department of Electrical Engineering and Automation,
Aalto University,
P. O. Box 13000,FI-00076 Aalto, Finland
viclial@ieee.org

George Nikolakopoulos
Computer Science, Electrical and Space Engineering Dept.
Luleå University of Technology
Luleå, Sweden
geonik@ltu.se

Abstract— **Implementation of unsupervised induction motor condition monitoring systems has drawn an increasing attention recently among motor drives manufacturers. In the case of soft-starters the possibility of incorporating fault detection features to their conventional functions provides an added value to those elements. Design and development of advanced algorithms that are able to automatically detect and alert about possible failures without requiring continuous human inspection is a challenging research goal. In this paper, an algorithm for the automatic detection of rotor damages in induction motors in the case of soft starting is proposed. The twofold approach relies, first, on the application of a time-frequency transform to the starting current signal and, second, on a pattern recognition stage based on the treatment of the time-frequency representation as a symbolic sequence. The innovation of this work is the implementation of the proposed approach for the automatic detection of rotor cage faults in soft-started motors. The experimental results prove the usefulness of the approach for the automatic detection of such faults and its potential for possible future implementation in soft-started machines.**

Keywords— broken bar fault, boft starters, symbolic time series analysis, time-frequency analysis

I. INTRODUCTION

Early fault detection in induction motors is a matter of high concern for industrial users, due to the extensive utilization of these machines in a wide variety of processes [1]. As a consequence, there has been an increasing research effort in the development of advanced fault diagnosis techniques that are able to monitor reliably the condition of such machines. A recent trend is to embed these techniques in the hardware of elements such as frequency converters or soft-starters, whose primary functions are not to determine the motor conditions. However, the incorporation of these techniques would provide an added value to these elements, a fact that is particularly relevant due to the increasing participation of frequency inverters and soft-starters in many industrial applications.

In this context, two factors play a crucial role: 1) the use of non-invasive quantities as a base for developing diagnosis techniques and 2) the adoption of unsupervised algorithms that do not require the user's constant presence during the decision making process. Regarding the first factor, techniques relying on the analysis of stator-currents have proliferated over recent years; these techniques are going beyond the conventional analysis of the steady-state current (Motor Current Signature Analysis, (MCSA) [1]).

Indeed, the most recent current-based methods rely on the application of advanced signal processing methods (based on time-frequency transformations) to the stator current regardless of the operation regime of the machine (including both transient or steady-state currents) [2-5]. These approaches enable both visualization of the frequencies of the fault components as well as their evolution over time. Thus, they obtain very characteristic signatures that enable to identify the fault with high reliability. As an example, in the event of a rotor cage fault, the most relevant component, amplified by the failure (the Lower Sideband Harmonic, (LSH)) shows a very particular time-frequency evolution during a direct startup that was well described in relevant works [2]. This evolution proved to be also present for the case of soft-started motors with damaged rotor cages [6].

Regarding the second issue, the automation without any user intervention for determining the presence or not of a fault, it is a major requirement for the implementation of such diagnosis techniques in the aforementioned devices. Nonetheless, most of the existing advanced current-based techniques hitherto developed still rely on the user expertness for the interpretation of the resulting time-frequency maps. They require the user to identify the components evolutions and decide if the fault is present or not. Some few works have dealt with the automation of this process, though all of them are restricted to line-started motors [7-9]. Moreover very few works deal with the problem of machine operating under soft-starter conditions [10-12].

This paper proposes an intelligent fault diagnosis algorithm that intends to assess the rotor condition in an automatic way, avoiding the necessity of continuous user supervision. Unlike other works, the algorithm is applied to the case of soft-started motors. Though the probability of rotor failure is supposed to be lower for this type of starting method, some real cases have been reported recently. The proposed method is based on two stages: first, the application of the Short Time Fourier Transform (STFT) to obtain characteristic signatures of the fault in the time-frequency plane. Afterwards, an intelligent algorithm based on the conversion of the two dimensional time-frequency representation into a one dimensional symbolic time series is applied. This symbolic time series is then turned into a "bag of patterns" representation that eventually feeds a nearest neighbor classifier to achieve automatic alerting on the presence of a fault and the estimation of its severity.

The results, obtained with laboratory experiments prove the effectiveness of the approach for the diagnosis of the rotor condition of soft-started squirrel-cage motors. This confirms its potential for a possible future implementation in these types of devices for fault detection monitoring purposes.

The rest of the paper is structured as follows: Section II describes the proposed integrated method and Section III presents the experimental set-up and the results. Finally section IV concludes this research work and discusses future directions.

II. The Method

The proposed integrated approach is based on a series of stages for the analysis of the stator current of an induction machine during soft starting: Isolation of the transient, application of the STFT to come up with a two dimensional representation, transformation of the two-dimensional representation into a one dimensional symbolic representation involving a modified version of Symbolic Aggregate ApproXimation (SAX), extraction of feature vectors from the symbolic sequence using the "bag of pattern" approach and finally the utilization of a nearest neighbor classifier for the fault diagnosis.

A. *Transient Isolation*

The proposed method is used during the transient of soft-starting in order to take advantage of the merits of the transient MCSA approach. To avoid the inclusion of the steady-state regime, a steady state detector is applied to signal the end of the transient [7]. The detector operates using a sliding window over which the Root Mean Square (RMS) value of the start-up line current is calculated. Then a second sliding window is used that operates over the sequence created during the previous step. This second window is used to estimate the standard deviation of the created time sequence and once its value falls below a predefined threshold the end of the transient regime is signaled.

B. *Time Frequency Representation*

During a motor starting at a constant rated frequency (direct start-up or soft starting) an asymmetry created by the breakage of a rotor bar is manifested through a frequency component that has a characteristic V pattern in the time frequency plane bellow the supply frequency as it is clearly indicated by equation 1 (considering the $-2\cdot k\cdot s$ case)

$$f_b=\left(1\pm 2\cdot k\cdot s\right)f_s,\qquad k=1,2,... \tag{1}$$

This V pattern is depicted in the time-frequency plane produced by the application of the STFT (Fig. 1). STFT is the most commonly encountered time-frequency transformation of a signal in engineering practice. It simply consists of the application of the Fourier transform over a sliding window $w(t)$ applied on the signal of interest $x(t)$.

$$X(t,\omega)=\int_{-\infty}^{\infty}x(\tau)w(\tau-t)e^{-j\omega\tau}d\tau \tag{2}$$

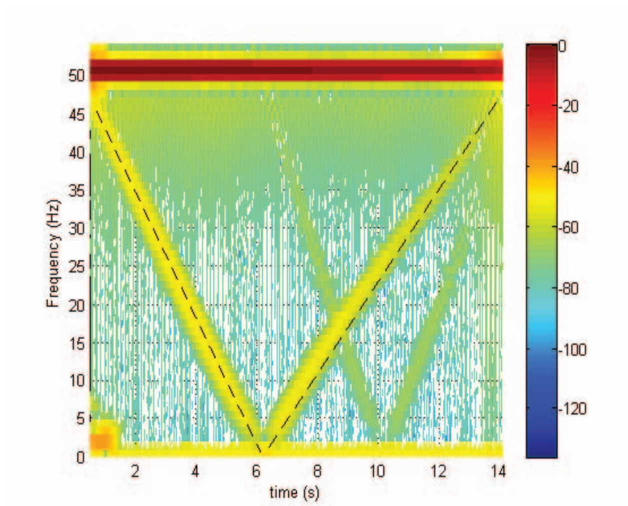

Fig. 1. The spectrogram of the start-up current for a direct on-line machine with rotor assymetry and its characteristic V pattern depicted in the spectrogram.

However due to the presence of additional time harmonics caused by the operation of the soft-starter as well as their interaction with inherent asymmetries, a wealth of other components appear making the isolation of the specific component quite difficult as it can be seen in Fig. 2. Therefore further processing is needed. One option is to try to isolate the specific component. The other option, which is pursued in this paper, is to process the whole time-frequency plane, or to be more specific, the part that is known to contain the characteristic frequencies, to come up with a more tractable/condensed representation where a conventional classification method can be applied.

C. SAX – Bag of Patterns Representation

The output of the STFT stage, the spectrogram, is a high-dimensional representation with varying dimensions depending on the duration of the start-up transient. In order to tackle both problems, a slightly modified version of the SAX algorithm [13] is applied as it will be explained in Section III. The standard SAX algorithm was developed for one dimensional time series and has already been used for the detection of broken bars in line fed machines [14]. SAX involves the following steps:

i) Normalization of the time series to have zero mean and standard deviation equal to one. By doing so most real life time series will end up having a Gaussian distribution

ii) Application of the Piecewise Approximate Aggregation (PAA) which reduces the dimensionality of the normalized time series $x = \{x[1], x[2], \ldots, x[N]\}$ of an arbitrary length N by dividing it into M ($M < N$) equally sized frames and taking the average value of the points falling within those frames [15], [16].

$$\overline{x}[i] = \frac{M}{N} \sum_{j=\frac{N}{M}(i-1)+1}^{\frac{N}{M}i} x[j] \text{, for } i = 1,2,...,M \tag{3}$$

Note: In case that N cannot be divided exactly by M the following formula can be used (which reduces to Eq. 3 when N can be divided exactly by M) [17][18].

$$\overline{x}[i] = \frac{M}{N}\left(\left(\left(\frac{N}{M}i+1\right)-\left\lfloor\frac{N}{M}i+1\right\rfloor\right)\cdot x\left[\left\lfloor\frac{N}{M}(i)+1\right\rfloor\right]\right)$$
$$+\frac{M}{N}\left(\sum_{j=\left\lfloor\frac{N}{M}(i-1)+1\right\rfloor+1}^{\left\lfloor\frac{N}{M}i+1\right\rfloor-1} x[j]\right)$$
$$+\frac{M}{N}\left(\left(1-\left(\left(\frac{N}{M}(i-1)+1\right)-\left\lfloor\frac{N}{M}(i-1)+1\right\rfloor\right)\right)\cdot x\left[\left\lfloor\frac{N}{M}(i-1)+1\right\rfloor\right]\right)$$

for $i = 1,2,...,M$ (4)

iii) Symbolization/discretization by creating a partition of the transformed space assuming Gaussian distribution of values after the normalization and thus selecting as “break points” the values that will produce equal-sized areas under a Gaussian curve. (Note: the assumption of a Gaussian distribution is not crucial [19] and it can be violated, but neither that nor the equiprobability violation [20] is so important when dealing with classification problems).

After the aforementioned procedures the output of symbols sequence is still a quite long one. At this point the “bag of patterns” representation proposed in [21] can be used to transform the symbolic sequence to a real valued feature vector.

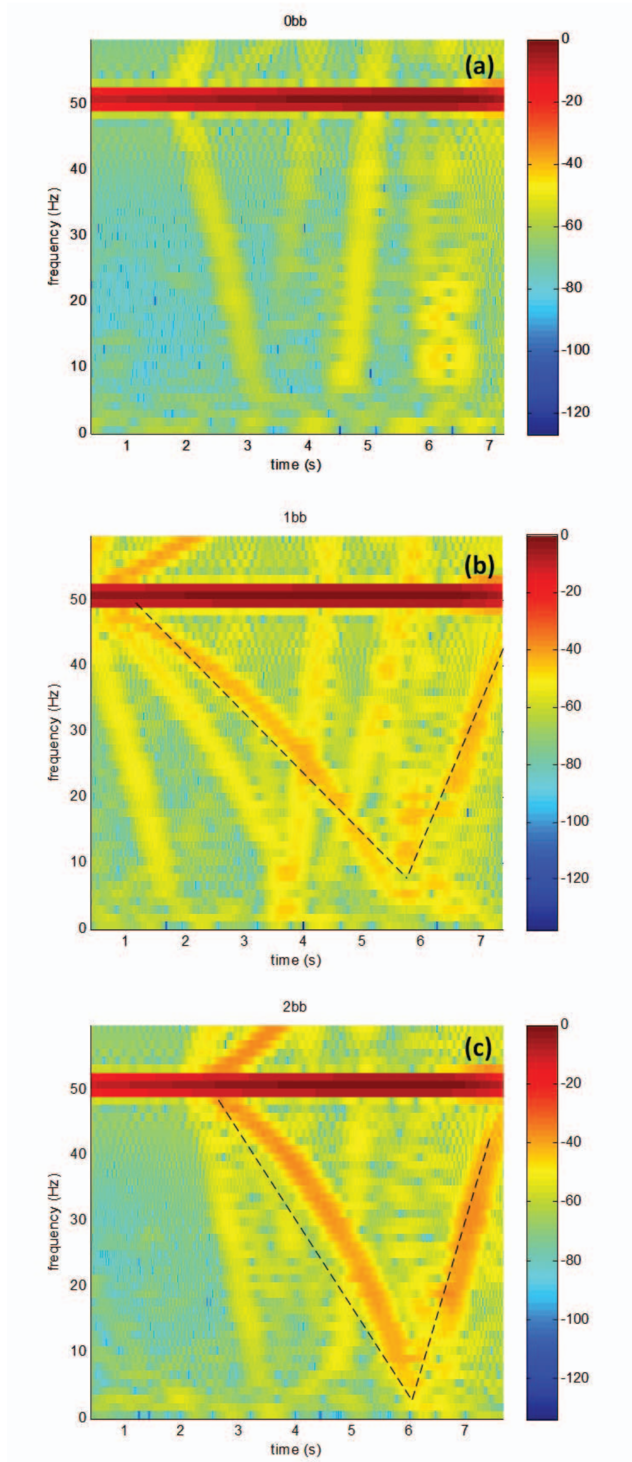

Fig. 2. The spectrogram of the start-up current for a machine with a) zero broken bars, b) one broken bar and c) with two broken bars.

In the original formulation of the "bag-of-patterns" feature extraction, a sliding window over the real valued time series, defines the limits of the word with the symbols/letters of the word given after the application of SAX within the sliding window. In this work a modified version was applied as is explained in the following section.

The steps that lead to a real valued feature vector are meant to be applied on a one dimensional time series. However the spectrogram is a two dimensional representation. In this work, the transformation of the $k \times l$ matrix (with the first dimension corresponding to frequency) of the spectrogram to a one dimensional representation is performed by simply concatenating all the rows together creating a $1 \times (k \cdot l)$ vector.

D. Classification and Diagnosis

The classification task and therefore the diagnosis is performed by a nearest neighbor classifier, which is the simplest member of the *K*-nearest neighbor (*K*-nn) family (*K*=1 for the nearest neighbor classifier). The diagnosis is performed by assigning the feature vector to one of three predefined classes /conditions (normal – one broken bar – two broken bars). *K*-nn classifiers belong to the family of memory-based classifiers and they are also referred as lazy learners since they do not use any training algorithm but rather rely on the training data that are stored in memory. They can create non-linear boundaries and are quite simple to implement.

In the nn classifier framework, the assignment of a new case $\mathbf{x}$ is performed by retrieving from the training data set the feature vector $\mathbf{x}_i$, as well as its corresponding label c_i, that is more similar to the unknown/new feature vector [22]. Similarity is usually measured using the Euclidian distance; the more similar two vectors are the smaller their Euclidian distance:

$$i = \arg\min_j \left\| \mathbf{x} - \mathbf{x}_j \right\| \tag{5}$$

E. Multidimensional Scaling

"Seeing is believing" [23]: in order to get a better understanding of the success (or failure) of a particular classification scheme it is useful to have a high level overview of the feature/attribute space. On the other hand, this is not always possible because the dimensionality of the feature space is much higher than three that is the number of dimensions that can be perceived by human vision. Therefore in order to be able to visualize high dimensional feature spaces, dimensionality reduction is applied [23], [24].

Dimensionality reduction is a very active field of research with many algorithms proposed over the past few years [24], [25]. Nevertheless as in most cases in life simpler methods can be quite competitive when it comes to real life problems [25]. Multidimensional Scaling (MDS) is a popular set of techniques [26] for dimensionality reduction, both of linear and non-linear nature. Classical MDS is a linear technique which is closely related to Principal Component Analysis (PCA). MDS methods try to retain the pairwise distances in the projected space $\mathbf{Y}$ as much as possible compared to the pairwise distances at the original space $\mathbf{X}$. That is achieved through the optimization (minimization) of the following stress function:

$$Str_{classicalMDS}(\mathbf{Y}) = \sum_i \sum_j \left(\left\| \mathbf{x}_i - \mathbf{x}_j \right\| - \left\| \mathbf{y}_i - \mathbf{y}_j \right\| \right)^2 \tag{6}$$

where, $\mathbf{x}_i$ ($i = 1, 2, ..., N$) are the original data points having a dimension of D, and $\mathbf{y}_i$ ($i = 1, 2, ..., N$) their corresponding projections having a dimension of , d $d < D$ (usually $d \ll D$).

III. Experimental Evalutation

For the validation of the proposed method a 1.1 kW induction motor coupled to a DC machine (Fig. 3) acting as a load is used [9]. For the case of the broken rotor bar(s) the breakages were artificially generated in the laboratory by drilling a hole in the junction point between the corresponding bar and the short-circuit end-ring. Three different motor conditions, (healthy, one broken bar and two broken bars) under voltage ramp soft start were tested with four signals acquired for each condition. The sampling frequency was set to 5 kHz for all experiments

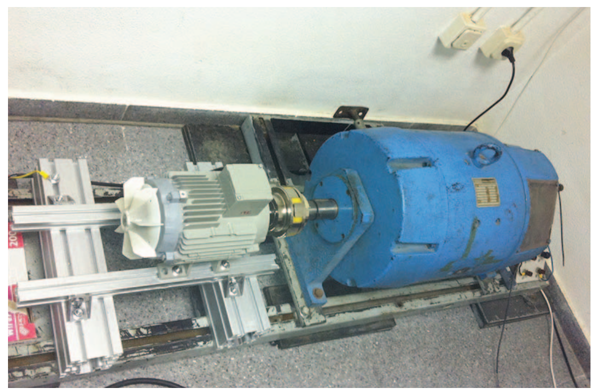

Fig. 3. The experimental set-up.

For the faults under study the amplitude of the corresponding frequency component, given by equation 1, determines the severity of the fault. Therefore, instead of applying the normalization step locally or to each case separately, the signals are globally normalized in such a way that the whole training set has zero mean and standard deviation equal to one. After normalization, the PAA is applied transforming the $1 \times (k \cdot l)$ vector to $1 \times (k \cdot 100)$ (in other words, the time axis is divided into 100 segments). In this way each case eventually ends up having the same length. Following that stage, symbolization takes place and the "bag of patterns" representation is applied by treating the resulting sequence as text using again a sliding window approach. It must be mentioned that since we are interested in the lower part of the time frequency plane only frequencies below 45Hz were considered. The whole procedure is illustrated at Fig. 4 just for the first "row" (the first frequency "bin" bellow the 45Hz limit) of the spectrogram corresponding to a machine with one broken bar.

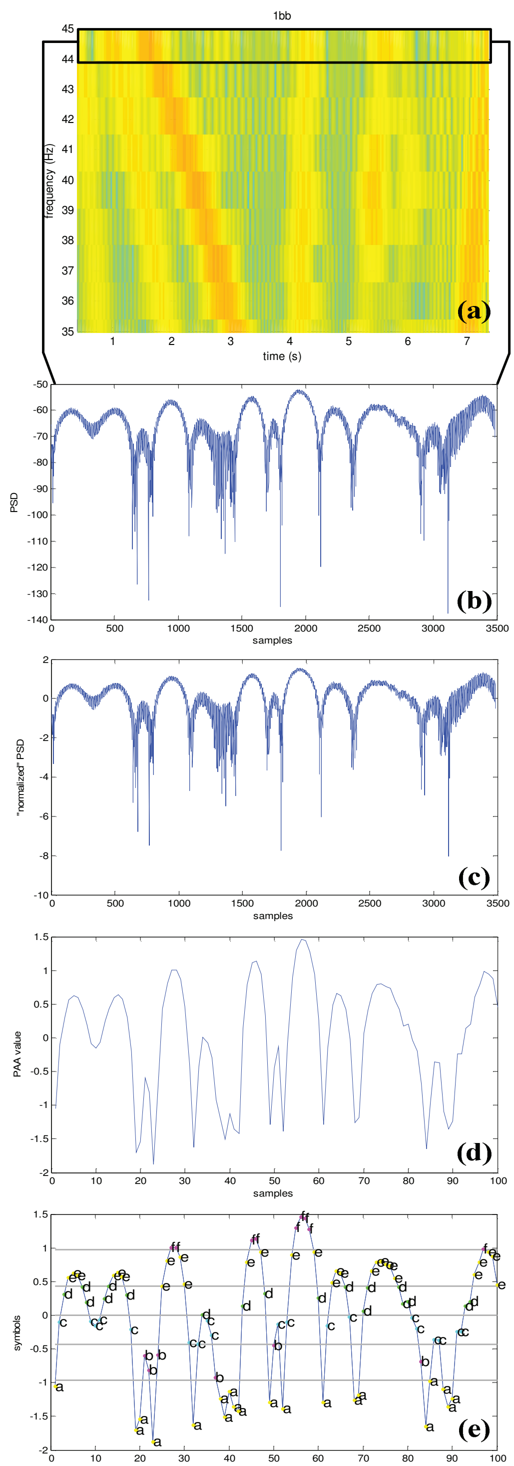

Fig. 4. a) a zoom in of the upper part of the spectrogram of a onebroken bar machine b) the one dimensional illustration of the first row of the spectrogram (upper (fixed) frequency bin and varying time), c) the normalization of the original signal to have zero mean and standard deviation equal to one d) the output of the PAA stage which reduces the number of samples to 100 and e) the output of the symbolization procedure which creates a SAX string – the thick horizontal lines correspond to the "break points".

From the last plot of Fig 4, we can see that the output of the SAX procedure is a string of the form acdeeeddccc…. In the "bag-of-pattern" representation the occurrences of the different words (in our case words of length 2) aa, ab, ac,…,ba, bb, …,ff, are counted using a sliding window (of length 2). The frequencies of occurrence of these words consists the "bag of patterns" representation to be used as feature vector by the classifier.

The validity of the method was tested using the leave one out (loo) procedure: each time one signal was left out for testing and the rest were used for training [22]. For each condition (healthy- one broken bar- two broken bars) four cases are used (12 cases in total).

In this work a simple configuration with a small word length (2) and a medium size alphabet (6) was tested without an exhaustive search for the optimal values for these parameters. Perfect discrimination was achieved as can be seen in Table I. An explanation of the success of the proposed method can be given by inspecting the lower 2-dimensional scatter plot of Fig. 5 depicting the projection of the future space into 2D using classical MDS [26]. As it can be seen the normal cases are concentrated quite far apart from the faulty ones. However the faulty ones are quite close revealing that further investigation is needed to come up with an optimal quantification scheme of the severity of the fault.

TABLE I. CONFUSION MATRIX – EXPERIMENTAL RESULTS

		Estimated class		
		0BB	***1 BB***	***2 BB***
True class	***0BB***	4	0	0
	1BB	0	4	0
	2BB	0	0	4

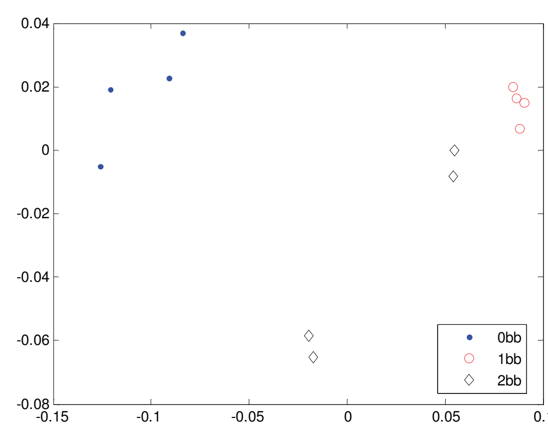

Fig. 5. Projection of the feature vectors of the 12 cases into a 2 dimensional space

IV. CONCLUSIONS

In this work, an automatic method for signaling the detection of broken rotor bars without the need for continues monitoring of an expert user, using the measured current of induction machines during soft starting was proposed. This is one of the very few studies involving the use of measurements coming from motors operating with soft starters. The method utilizes a very simple time frequency technique and then a quite advanced method which treats the two dimensional representation as a long string of symbols. The creation of a symbolic representation naturally leads to the adoption of tools from the field of information retrieval and text mining for its further processing and categorization. The results indicate that the method is promising creating quite distinct representations between normal and faulty situations. This is revealed by the application of MDS.

In future work the method will be also tested for soft starting with a current limiter before safer conclusions can be drawn. Further testing also is needed in order to find the optimum value for the parameters involved during the feature extraction stage as well as their robustness against measurements coming from other motors.

Acknowledgment

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