

ACTIVATION HEBBIAN LEARNING RULE FOR FUZZY COGNITIVE MAPS

Elpiniki Papageorgiou, Chrysostomos D. Stylios and Peter P. Groumpos

*Laboratory for Automation and Robotics
Department of Electrical and Computer Engineering
University of Patras, Rion 26500, GREECE
Tel. +30 61 997293, Fax. +30 61 997309 groumpos@ee.upatras.gr*

Abstract: Fuzzy Cognitive Maps (FCM) is a soft computing, modeling methodology for complex systems, which is originated from the combination of fuzzy logic and neural networks. Many different learning algorithms have been suggested for the training of neural networks. Only some initial thoughts on learning rules have described for FCMs. A learning law is a mathematical algorithm, which can train the FCM by selecting the appropriate weights and it is very important for a system to have learning and adaptive capabilities. In this paper a new learning algorithm, the Activation Hebbian Learning (AHL) has been proposed for FCMs. The learning rule for a FCM is a procedure where FCM weight matrix is modified in order the FCM to model the behavior of a system. Simulation results proving the strength of the learning rule are provided. *Copyright © 2002 IFAC*

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1. INTRODUCTION TO FCMs

Fuzzy Cognitive Maps (FCM) is a soft computing approach originated from the hybrid implementation of fuzzy logic techniques to represent knowledge and behaviour of a system using a network of interconnected nodes. Nodes of Fuzzy Cognitive Map represent concepts of the model of the system. FCM is consisted of concepts that represent the dynamic behaviour of a simple and/or complex system. FCMs aim to emulate human reasoning, they capture and emulate the nature of human being in describing, presenting and modelling systems, including tolerance for imprecision and granulation of information (Stylios and Groumpos, 2000).

Fuzzy Cognitive Maps could be seen as fuzzy signed graphs with feedback. They consist of nodes-concepts C_i and weighted interconnections w_{ij} between concept C_i and concept C_j . A FCM models a dynamic complex system as a collection of concepts and cause and effect relations between concepts. A simple illustrative picture of a Fuzzy Cognitive Map is depicted in Fig. 1, consisted of five nodes-concepts.

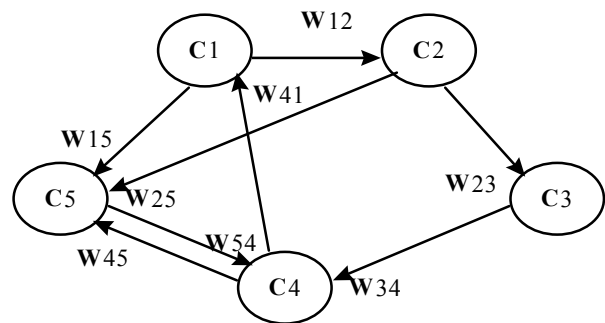


Figure 1. A simple Fuzzy Cognitive Map

The weight w_{ij} describes the grade of causality between concepts C_i and C_j it takes values in the interval $[-1,1]$. The sign of the weight indicates positive causality $w_{ij} > 0$ between concept C_i and concept C_j , which means that an increase in the value of concept C_i will cause an increase in the value of concept C_j and a decrease in the value of concept C_i will cause a decrease in the value of concept C_j . When there is negative causality between two concepts, then $w_{ij} < 0$; the increase in

the value of the first concept means the decrease in the value of the second concept and the decrease of concept C_i causes the increase in value of C_j . When there is no relationship between concepts, then $w_{ij}=0$. The strength of the weight w_{ij} indicates the degree of influence between concept C_i and C_j .

Generally, the value of each concept is calculated, computing the influence of other concepts to the specific concept, by applying the calculation rule of equation (1):

$$A_i(t+1) = f(A_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^n A_j(t)w_{ji}) \quad (1)$$

Where $A_i(t)$ is the value of concept C_i at time t , $A_j(t-1)$ is the value of concept C_j at time $t-1$, w_{ji} is the weight for the interconnection between concept C_j and concept C_i and f is the sigmoid function.

The methodologies for development and construction of Fuzzy Cognitive Map have great importance for FCMs utilization in the modelling of complex systems. FCM represents the human knowledge on the operation of the system and experts develop FCMs using their experience and knowledge on the system. Construction methodologies rely on the exploitation of experts' experience on system's model and behaviour. Experts determine the number and kind of concepts and the interrelationships among concepts (Stylios and Groumpos, 1999).

It must be mentioned that the use of experts is very critical in the designing and development of Fuzzy Cognitive Maps. Experts who have knowledge and experience on the operation and behavior of the system are involved in the determination of concepts, interconnections and assigning casual fuzzy weights to the interconnections (Kosko, 1992).

2. UNSUPERVISED LEARNING METHODS

Generally, Learning Rules for a connectionist system are algorithms or methodologies that govern changes in the weights of the connections in a network. One of the simplest learning procedures for two-layer networks is the Hebbian learning rule, based on a rule initially proposed by Hebb (Hebb,1949). Hebb states that the simultaneous excitation of two neuron results in the strengthening of the connections between them. Hebb's postulate of learning states:

1.If two neurons of each side of the synapse are activated synchronously (simultaneously), then the strength of that synapse is selectively increased.

2.If two neurons on either side of a synapse are activated asynchronously, then that synapse is selectively weakened or eliminated.

Supervised Learning Rules are learning rules, which incorporate an error reduction procedure or error correction procedure (delta rule, back propagation). Learning rules, which incorporate an error reduction procedure, utilize the discrepancy between the desired output pattern and an actual output pattern to change (improve) its weights during training. A Learning Rule is typically applied repeatedly during the training period to the same set of training inputs across a large number of training loops with error gradually reduced across training, as the weights are fine-tuned.

In unsupervised learning there is no training set of data or feedback from the environment to indicate what the outputs of a network should be or whether they are correct. The network finds by itself any relationships that may exist in the input data and translate the discovered relationship into outputs (Lin, *et al.*, 1996). In this section four typical unsupervised learning rules will be described: signal Hebbian, competitive Hebbian, differential Hebbian, and differential competitive.

The unsupervised signal Hebbian learning rule, (Lin, *et al.*, 1996) is based on increasing the weight w_{ij} between a neuron's input x_j and output y_i in proportion to their product:

$$w_{ij}(k+1) = w_{ij}(k) + \alpha \cdot y_i(k) \cdot x_j(k) \quad (2)$$

where α is a positive number called the learning constant, which determines the learning rate.

The Differential Hebbian Learning rule which uses time derivatives (Kosko, 1997), and its expression for discrete type mode is:

$$w_{ij}(k+1) = w_{ij}(k) + \alpha \cdot y_i(k) \cdot x_j(k) + \dot{y}_i(k) \cdot \dot{x}_j(k) \quad (3)$$

The competitive Hebbian Learning Rule is known as Grosseberg's learning rule (Zurada, 1992) and it can be expressed by the equation:

$$w_{ij}(k+1) = s_i(y_i) \cdot [s_j(x_j) - w_{ij}(k)] \quad (4)$$

where $s_i(y_i) = \frac{1}{1 + \exp(-c \cdot y_i)}$ and $c > 0$.

The differential competitive learning rule combines competitive and differential Hebbian Learning Rules and has the following expression:

$$w_{ij}(k+1) = w_{ij}(k) + \Delta s_i(y_i(k)) \cdot (x_j(k) - w_{ij}(k)) \quad (5)$$

where $\Delta s_i(y_i(k))$ is the difference of signals of the i th neuron.

2.1 Unsupervised learning for FCMs

Kosko used the Differential Hebbian Learning (DHL) for unsupervised learning for a special case the bivalent FCMs (Kosko, 1997). The DHL law correlates the changes of two concepts. If concept A moves in the same direction with concept B (e.g. B increases when A increases), the edge strength w_{ij} between the two concepts is increased; otherwise the edge strength is decreased.

The training data resembles a sequence of state vectors. Training is done by going through each state vector and modifying the FCM matrix based on the DHL law. At each time step t , the value for w_{ij} , the edge linking concept C_i and concept C_j , is given by the discrete version of the DHL law:

$$w_{ij}(t+1) = w_{ij}(t) + \mu_t (\Delta C_i(t) \cdot \Delta C_j(t) - w_{ij}(t)) \quad (6)$$

where ΔC_i is the change in the value of i th concept and $\Delta C_i(t) = C_i(t) - C_i(t-1)$. The learning coefficient μ_t decreases slowly over time, with the following equation (Kosko, 1992):

$$\mu_t = 0.1 \left[1 - \frac{t}{1.1N} \right] \quad (7)$$

where the constant N ensures the learning coefficient μ_t never becomes negative.

Differential Hebbian Learning was proposed to make FCMs adapt causal link strengths as part of an unsupervised training with a sequence of state vectors. However, there is no guarantee that DHL will encode the sequence into the FCM.

3. ACTIVATION HEBBIAN LEARNING (AHL)

Generally, there are three recall modes that can be used to apply a learning rule: asynchronous updating, synchronous updating and sequential updating. The unsupervised learning rule proposed by Kosko has been used exclusively for synchronous updating modes between neurons and in some cases have been used for sequential updating modes.

The nature of FCMs encourages us to examine and propose the usage of asynchronous updating mode for the weights of the FCM. This is suggested because the values of concepts in the FCM are not updated at the same activated step and there is not been taken into consideration any time relationship between concepts. Asynchronous updating for weights is proposed here for the training of FCMs and the calculation of the desired values of concepts.

The asynchronous mode suggests that each time there is only one activation concept at the FCM. The new value of the activation concept acts as trigger, which

causes updating of the weights of the connections between the activation concept and the others.

It should be noticed here that according to the infrastructure of the Fuzzy Cognitive Map, the expert chooses the activation concept at every step. So, the sequence of activation steps between concepts in FCM is suggested and defined by experts who develop the FCM according to the relationships between concepts and the most important factors, which affect the desired value of concept (DVC).

A new training rule is proposed here for first time based on the general unsupervised Hebbian learning algorithm. This new algorithm will be referred as the Activation Hebbian Learning (AHL) and has the following discrete type asynchronous mode:

$$w_{ij}(t+1) = \alpha \cdot w_{ij}(t) + \eta(t) \cdot A_i^{act}(t) \cdot A_j(t) \quad (8)$$

It is supposed that concept A_i is the activation concept and there is an iteration t . A_i^{act} is the value of the activation concept A_i on the iteration t and A_j is the value of interconnected concept at the same iteration. The coefficients α and η are positive parameters, adjusting the values of the weight matrix, with $\alpha > \eta$, taking values $0 < \alpha < 1$ and $0 < \eta < 1$.

According to the established procedure of constructing FCMs experts define the FCM and suggest the weights w_{ij} . It is desirable to keep in mind their suggestions. Thus the coefficient α is suggested to take values near to 1 but it never becomes equal to it. Coefficient η is the learning rate coefficient and the parameter that affects the concepts at the iteration t and influences the values of weights. After some iterations, we may want to eliminate the influence of A_i^{act} on $w_{ij}(t+1)$, so it is proposed that the coefficient decreases exponentially and finally, after some iterations, takes very small values (but never zero). Thus, the contribution of coefficient α in the modification of weights is more significant than η .

The learning rate coefficient can be expressed as:

$$\eta(t) = k \cdot \exp(-\lambda \cdot t) \quad (9)$$

where k is a constant coefficient with value 0.02 that has been proven to be the best choice after many simulation experiments. The parameter λ takes values that make the value of η to decrease slowly after a number of steps and no sharply attenuation for the first 20 steps. The suggested values of λ are between 0.1-0.2 and the proposed one is 0.1.

Taking into account the previous discussion the proposed Activation Hebbian Learning (AHL) with $k=0.02$ and $\lambda=0.1$, has the following expression:

$$w_{ij}(t+1) = \alpha \cdot w_{ij}(t) + 0.02 \exp(-0.1 \cdot t) \cdot A_i^{act} \cdot A_j \quad (10)$$

The equation (1) which calculates the value of each concept is updating and takes the following form where the value of weight w_{ji} is calculated using equation (8):

$$A_i(t+1) = f(A_i(t) + \sum_{\substack{j \neq i \\ j=1}}^N w_{ji}(t+1) \cdot A_j(t)) \quad (11)$$

This rule considers the asynchronous updating of events between concepts. In each iteration there is one or more updated values of the related concepts that are defined by the users.

Experts describe FCMs and determine the weights using a fuzzy logic approach with linguistic values to describe the causal relationship among concepts (Stylios and Groumpos, 2000). Furthermore, the initial values of concepts influence the modification of values of weights because of the unsupervised training method. This is shown in equation (8) where the parameter η is multiplied with the values of concepts A_i, A_j calculated at previous iteration t .

4. A PROCESS CONTROL EXAMPLE

In this section the new proposed Activation Hebbian Learning (AHL) is implemented to train the FCM that models a simple process control problem.

A simple process example is considered where there is one tank and three valves that influence the amount of liquid in the tank; figure 2 shows an illustration of the system. Valve1 and valve2 empty two different kinds of liquid into tank1 and during the mixing of the two liquids a chemical reaction takes place into the tank. A sensor is located inside the tank and it measures the specific gravity of the liquid that is produced into tank, by the mixing of the two incoming liquids. When value of specific gravity lies in the range between (G_{max}) and (G_{min}), this means that the desired liquid has been produced in tank. Moreover, there is a limit on the height of liquid in tank, which cannot exceed an upper limit (H_{max}) and a lower limit (H_{min}). So the control target is to keep these variables in the range of values:

$$G_{min} \leq G \leq G_{max}$$

$$H_{min} \leq H \leq H_{max}$$

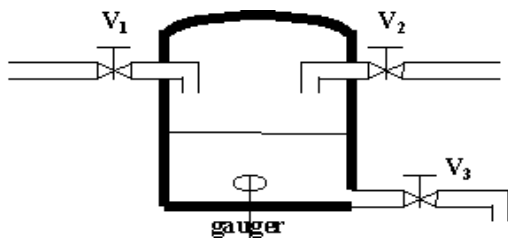


Fig. 2. The illustration for simple process example

A Fuzzy Cognitive Map that models and controls this system is developed and depicted on fig. 3. Three experts followed a methodology (Stylios and Groumpos, 2000) constructed the FCM. They jointly determined the concepts of the FCM and then each expert drawn the interconnections among concepts and he assigned a weight for each interconnection.

The FCM is consisted of five concepts that are determined as follows:

- Concept 1 – the amount of the liquid that Tank 1 contains is depended on the operational state of Valves 1, 2 and 3;
- Concept 2 – the state of Valve 1 (it may be closed, open or partially opened);
- Concept 3 - the state of Valve 2 (it may be closed, open or partially opened);
- Concept 4 - the state of Valve 3 (it may be closed, open or partially opened);
- Concept 5 –the specific gravity of the liquid into the tank.

Experts have suggested the appropriate weights of the FCM that are shown in the following weight matrix:

$$w^{initial} = \begin{bmatrix} 0 & -0,4 & -0,25 & 0 & 0,3 \\ 0,36 & 0 & 0 & 0 & 0 \\ 0,45 & 0 & 0 & 0 & 0 \\ -0,9 & 0 & 0 & 0 & 0 \\ 0 & 0,6 & 0 & 0,3 & 0 \end{bmatrix}$$

5. TRAINING THE FCM OF PROCESS EXAMPLE WITH AHL

Now the proposed learning rule will be applied to train the FCM developed in the previous section and determine the final weight matrix. In this section, the training and the implementation phase will be described, examined and compared. In the first phase, experts propose the initial values of concepts and a trained weight matrix is derived implementing the suggested AHL rule. At the second phase, random initial values of concepts are considered and the derived weight matrix from first case is used as the initial weight matrix for the next training procedure, calculating the Desired Values of Concepts (DVCs). It is proved that using as weight matrix the derived from the training phase, the FCM always converge to the same region of concept value for any initial concept values.

The weights of the interconnections are calculated according to Activation Hebbian Learning (AHL) based on equation (10) where the coefficient α has chosen to have a constant value 0.98. Equation (11) calculates the updated values of concepts based on asynchronous updating mode.

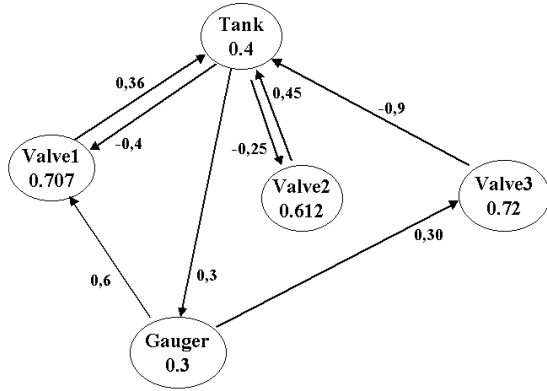


Fig. 3. The FCM model of the process

The first assumption for this asynchronous learning rule is the determination of the activated concept A^{act} and the sequence of activation. The activated concept C_1 which represents the height of water in the tank, affects the subsequent values of interconnected concepts and causes new activated values. These activated values are compared for each concept to their previous values. This procedure is called cycle and results in the calculation of new values for concepts and weights. The cycles stop when the concepts have reached the Desired Values of Concepts (DVCs) and then the final weight matrix is calculated.

In this example each cycle consists of 3 steps where the 2nd step is consisted of 4 subsequent steps. During each subsequent step values of some concepts are calculated. The number of subsequent steps is case dependent. The experts determine the number of subsequent steps based on the specific problem; in our case they choose three steps:

Step 1:

The initial values of nodes A_0 and the matrix of weights w_0 are given by experts.

Step 2:

The equation (11) is used to calculate the activated concepts and the equation (10) the new values of weights after training. This step consists of 4 subsequent steps for the proposed practical process control problem.

Sub. Step1: the value of activated concept C_1 is calculated using equation (11). Only the weighted arcs that exist between the activated concept and the concepts connected with it are calculated at this step, by equation (10).

Sub. Step 2: The activated concept C_1 , with its new value affects the sequence of concepts. The next concepts are concepts C_2 and C_3 because of their interconnections with the Activated C_1 . These two activated concepts and the derived weight matrix are calculated using equation (11) and (10) respectively. At this step also, the equation (10) is used for the calculation of the weighted arcs that exist between

the activated concepts and the other interconnected concepts with them.

Sub. Step 3: The activated concepts in Sub. step 2 also affect the next sequential concept, which is concept C_5 . At this activation step, the activated concept represents the gauger. The new matrix of weights of interrelated concepts is also derived from the learning rule.

Sub. Step 4: The activated concepts in the previous steps affect the last sequential concept C_4 . The value of concept is calculated according to equation (11) and the matrix of weights derives, after applying Activation Hebbian Learning Rule (equation 10).

Step 3:

The new calculated values of concepts are compared to their previous values. If the subsequent values of each concept deviate at a small amount, let's consider only after the 3rd decimal point, then the algorithm stops. A deviation on 3rd decimal point is considered acceptable and the Final Values of Concepts range in a desirable region of values. Otherwise if the consequence values vary more than acceptable, then the algorithm continues at step 2, until the final activated concepts do not change their values any more.

The experts suggest the following initial values of nodes: $A_0=[0.4, 0.7077, 0.612, 0.72, 0.3]$

Table 1 Values of Concepts for 12 cycles (48 steps)

Sub. Step	Tank (C1)	Valve1 (C2)	Valve2 (C3)	Valve3 (C4)	Gauger (C5)
1	0,5736				
2		0,6665	0,6236		
3					0,7119
4				0,5363	
5	0,6539				
6		0,7102	0,6360		
7					0,7870
8				0,5456	
9	0,6894				
10		0,7294	0,6481		
11					0,7988
12				0,5475	
13	0,7004				
14		0,7313	0,6545		
15					0,7975
16				0,5540	
17	0,7049				
18		0,7340	0,6621		
19					0,7955
20				0,5710	
21	0,7065				
22		0,7347	0,6675		
23					0,7911
24				0,5805	
25	0,7071				
26		0,7319	0,6709		
27					0,7841
28				0,5884	
29	0,7045				
30		0,7582	0,6724		

31				0,7775	
32			0,5969		
33	0,7014				
34		0,7240	0,6734		
35				0,7706	
36			0,6033		
37	0,6982				
38		0,7198	0,6749		
39				0,7638	
40			0,6087		
41	0,6951				
42		0,7155	0,6758		
43				0,7551	
44			0,6134		
45	0,6921				
46		0,7114	0,6772		
47				0,7516	
48	0,6921	0,7114	0,6772	0,6175	0,7516

Hence, activated concepts stop change their value the learning procedure has been accomplished and the derived matrix of weights is calculated after 48 steps (which means 12 cycles):

$$w^{AHL} = \begin{bmatrix} 0 & -0,1108 & -0,0576 & 0,0347 & 0,1534 \\ 0,1775 & 0 & 0,0410 & 0,0391 & 0,0431 \\ 0,2077 & 0,0391 & 0 & 0,035 & 0,0391 \\ -0,3066 & 0,0391 & 0,035 & 0 & 0,0355 \\ 0,04 & 0,2706 & 0,0391 & 0,1496 & 0 \end{bmatrix}$$

After training new values for weights are produced that describe new relationships among the 5 concepts. Thus, a new model for the process has been produced. It is noticeable that the initial zero weights no more exist, and new interconnections with new weights have been assigned. Only diagonal values remain equal to zero. This means that all concepts affect the related concepts, except themselves, and the weighed arcs show the degree of this relation and change. For example, the weighted arc w_{12} with initial value -0.4 , after 12 steps takes the value -0.1108 , which means that concept C_1 influences the concept C_2 (valve 1) less. Moreover, the initial zero value of weight w_{23} has changed and the derived value, after 12 cycles, is 0.04 , which means that the concept C_2 affects the concept C_3 finally. Thus, this training affects the dynamical behavior of the system.

For the second phase it is considered that the FCM has been trained and the corresponding weight matrix has been developed. Random initial values are used for the concepts and the weight matrix after training is used as the initial weight matrix for the implementation of AHL algorithm.

For a set of random initial values A^0 , the final values of concepts are calculated and presented at A^{act} :

$$A^0 = [0.1, 0.4, 0.3, 0.5, 0.2],$$

$$A^{act} = [0.6981, 0.7150, 0.6787, 0.6198, 0.7514]$$

These final values at second phase vary at the third decimal point compared to the values derived at first phase. This is an acceptable deviation between the desired values of concepts, as other parameters less important exist in an FCM and take part in the control process.

6. CONCLUSION

A new training methodology, the AHL rule for the weights of FCMs has been introduced, presented and tested for a real process control problem. Fuzzy Cognitive Maps were presented and the main learning algorithms for Neural Networks have been discussed.

It has been proved that using the AHL, a trained FCM has been determined, which exhibit almost stable point behavior. With the proposed algorithm the experts suggest the matrix of weights of the FCM, and then using the AHL rule a new weight matrix is derived that can be used for any set of initial values of concepts and the FCM always converge to the desired region of values. The stability of the system is still an open research problem

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